Grand Unified Models and Cosmology

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Declaration

Apart from chapter 1, in which background material is reviewed, and chapter 2, which is the result of work in collaboration with Dr Anne-Christine Davis, the work presented in this dissertation is my own and includes no material which is the outcome of work done in collaboration. The work is original, and has not been submitted for any other degree, diploma, or other qualification.

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Summary

The cosmological consequences of particle physics grand unified theories (GUTs) are studied. Cosmological models are implemented in realistic particle physics models. Models consistent from both particle physics and cosmological considerations are selected.

After a brief introduction to the big-bang cosmology and to the particle physics standard model, the ideas of grand unification, supersymmetry, topological defects and inflation are introduced. Special emphasis is given to the physics of massive neutrinos and to supersymmetric GUTs. The elastic and inelastic scattering of fermions off cosmic strings arising in a nonsupersymmetric SO(10) model are first studied. It is then shown that the presence of supersymmetry does not affect the conditions for topological defect formation. By studying the impact of the spontaneous symmetry breaking patterns from supersymmetric SO(10) down to the standard model on the standard cosmology, through the formation of topological defects, and by requiring that the model be consistent with proton lifetime measurements, it is shown that there are only three patterns consistent with observations. Using this analysis, a specific model is built. It gives rise to a false vacuum hybrid inflationary scenario which solves the monopole problem. At the end of inflation, cosmic strings form. It is argued that this type of inflationary scenario is generic in supersymmetric SO(10) models. Finally, a new mechanism for baryogenesis is described. This works in unified theories with rank greater or equal to five which contain an extra gauge $U(1)_{B-L}$ symmetry, with right-handed neutrinos and $B-L$ cosmic string, i.e. cosmic strings which form at the $B-L$ breaking scale, where B and L are baryon and lepton numbers.

KALIAYEV, *égaré.*

Je ne pouvais pas prévoir... Des enfants, des enfants surtout. As-tu regardé des enfants? Ce regard grave qu'ils ont parfois... Je n'ai jamais pu soutenir ce regard... Une seconde auparavant, pourtant, dans l'ombre, au coin de la petite place, j'étais heureux. Quand les lanternes de la calèche ont commencé à briller au loin, mon coeur s'est mit à battre de joie, je te le jure. Il battait de plus en plus fort à mesure que le roulement de la calèche grandissait. Il faisait tant de bruit en moi. J'avais envie de bondir. Je disais " oui, oui "...Tu comprends?

Il quitte Stepan du regard et reprend son attitude affaissée.

J'ai couru vers elle. C'est à ce moment que je les ai vus. Ils ne riaient pas, eux. Ils se tenaient tout droit et regardaient dans le vide. Comme ils avaient l'air triste! Perdus dans leurs habits de parade, les mains sur les cuisses, le buste raide de chaque côté de la portière! Je n'ai pas vu la grande duchesse. Je n'ai vu qu'eux. S'ils m'avaient regardé, je crois que j'aurais lancé la bombe. Pour éteindre au moins ce regard triste. Mais ils regardaient toujours devant eux.

Il lève les yeux vers les autres. Silence. Plus bas encore.

Alors je ne sais pas ce qui s'est passé. Mon bras est devenu faible. Mes jambes tremblaient. Une seconde après, il était trop tard. (*Silence. Il regarde à terre.*) Dora, ai-je rêvé, il m'a semblé que les cloches sonnaient à ce moment là?

Albert CAMUS, *Les justes*, ACTE II.

Contents

Chapter 1

Introduction

1.1 Particle physics and the standard cosmology

The particle physics standard model and theories beyond, such as grand unified theories or supersymmetry, are essential ingredients for any successful description of the evolution of the universe from very early time until today. Particle physics is particularly necessary for the understanding of the very early universe. It is also the only way to solve some cosmological problems, such as the baryon asymmetry of the universe or the dark matter problem. Particle physics also plays an important role in inflationary cosmology. This thesis is interested in this interplay between particle physics and cosmology.

In this introductory chapter, we briefly review the big-bang cosmology and the particle physics standard model and outline their main problems. We show the need for theories beyond these standard models, and introduce the ideas of grand unification, supersymmetry, topological defects and inflation. A general consequence of unified theories is that neutrinos acquire a mass. This has interesting cosmological consequences. We discuss the physics of massive neutrinos in Sec. 1.2. We compare Dirac and Majorana cases and review the see-saw mechanism. Supersymmetric grand unified theories (GUTs) have more predicted power than nonsupersymmetric ones. In Sec. 1.3, we discuss the main features of supersymmetric GUTs. We introduce the notions of superspace, superfields and superpotential, and explain how to construct supersymmetric Lagrangians. For a review of cosmology the reader is referred to Ref. [5], for quantum field theories to Ref. [6], for topological defects to Ref. [7], for grand unified theories to Ref. [8] and for supersymmetry to Ref. [9]. Throughout the manuscript, we will work in units where $\hbar = c = k_B = 1$. All quantities will be expressed in GeV $= 10^9$ eV such that

$$[Energy] = [Mass] = [Temperature] = [Length]^{-1} = [Time]^{-1}. \tag{1.1}$$

Some useful conversion factors are

$$\text{Temperature :} \quad 1\text{GeV} = 1.16 \times 10^{13}\text{K}$$

$$\begin{aligned} \text{Mass :} \quad & 1\text{GeV} = 1.78 \times 10^{-24}\text{g} \\ \text{Time :} \quad & 1\text{GeV}^{-1} = 6.58 \times 10^{-25}\text{sec} \\ \text{Length :} \quad & 1\text{GeV}^{-1} = 1.97 \times 10^{-14}\text{cm.} \end{aligned}$$

In these units, the Planck mass $M_{\rm pl} = 2.18 \times 10^{-5}\text{g} = 1.22 \times 10^{19}\text{GeV}$ and the gravitational constant $G = M_{\rm pl}^{-2}$.

The hot big-bang cosmology is based upon the Robertson-Walker metric which describes an homogeneous and isotropic universe. It can be written in the form

$$ds^2 = dt^2 - a(t)^2\Big\{\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2\Big\} \tag{1.2}$$

where (r, θ, ϕ) are spatial coordinates, referred to as comoving coordinates. $a(t)$ is the cosmic scale factor and k determines the geometry of space. With an appropriate rescaling of the parameters, k can be chosen to be $+1$, 0 or -1 corresponding to open, flat and closed geometries respectively. The value of k and the time dependence of the function a can be determined by solving Einstein's equations.

In an homogenous and isotropic cosmology the stress-energy tensor is taken to be that of a perfect fluid, $T_{\mu\nu} = diag(\rho, -p, -p, -p)$, where $\rho(t)$ and $p(t)$ are the energy and pressure densities respectively. Einstein's equation then lead to the Friedman equation

$$H^2 \equiv (\frac{\dot{a}}{a})^2 = \frac{8\pi G\rho}{3} - \frac{k}{a^2} + \frac{\Lambda}{3}, \tag{1.3}$$

where $H(t)$ is the Hubble parameter which characterises the expansion of the universe and Λ is the cosmological constant. The present-day value of the Hubble parameter $H_0 = 100h_0$ kms^{-1}Mpc^{-1} and observational bounds give $0.4 < h_0 < 1$.

Assuming no cosmological constant, it is convenient to define the 'critical' energy density,

$$\rho_c = \frac{3H^2}{8\pi G} = 1.88 \times 10^{-29} h^2 \quad \text{kgm}^{-3}. \tag{1.4}$$

Defining the density parameter

$$\Omega = \frac{\rho}{\rho_c}, \tag{1.5}$$

the Friedman equation (1.3) can be rewritten in the form

$$\frac{k}{H^2 a^2} \equiv \Omega - 1, \tag{1.6}$$

and hence $\Omega > 1, < 1, = 1$ for $k = +1, -1$,0. Ω_0 is a very difficult quantity to measure. Observational limits give $0.1 \leq \Omega_0 \leq 2$ [10], and hence observations do not tell us whether the universe is open, flat, or closed. Luminous matter (stars and associated material) contributes to Ω_0 by a very small amount, $\Omega_{Lum} \simeq 0.005$. Hence, since observations give $\Omega_0 \geq 0.1$, we deduce that the missing energy density must be in the form of non-luminous dark-matter.

Big-bang nucleosynthesis (BBN) predicts the abundances of the light elements He^3, D, He^4 and Li^7, with respect to hydrogen, the most abundant element in the universe, as a function of an adjustable parameter, the baryon-to-photon ratio, $\eta = \frac{n_B}{n_\gamma}$. BBN only works if, within the uncertainties, $\eta \simeq 2 \times 10^{-10} - 7 \times 10^{-10}$ [11]. From the constraints on the parameter η follows the constraints on the baryonic dark-matter energy density $\Omega_B h_0^2 \simeq 0.007 - 0.025$.

While a conservative limit for Ω_0 is $\Omega_0 \geq 0.1$, other observational limits give $\Omega_0 \geq 0.3$. $\Omega_0 \simeq 1$ is also favoured by theoretical arguments. These considerations, together with the limits on the baryonic dark matter energy density, strongly suggest that there must be some non-baryonic dark matter. Some non-baryonic dark-matter candidates are the neutrino, if massive [12], the lightest superparticle (LSP), if stable [13], and the axion [14]. A dark-matter candidate is called 'hot' if it was moving at relativistic speed at the time at which galaxies started to form, and it is called 'cold' if it was non-relativistic at that time. Massive neutrinos are hot whereas the LSP and the axion are cold dark-matter candidates.

The hot big-bang cosmology predicts the expansion of the universe and the present abundances of the light-elements. Its best recent success is the predicted perfect black-body spectrum of the cosmic background radiation (CBR) measured by COBE (Cosmic Background Explorer Satellite) [15]. But in spite of all its successes, the hot big-bang model faces problems and unanswered questions remain. Observational bounds give $0.1 \leq \Omega_0 \leq 2$, implying that Ω was incredibly close to unity from very early times until now. This is known as the flatness problem. Also, which process(es) lead to the baryon asymmetry of the universe? Which process(es) can predict the small ratio η? What is the nature of the dark-matter of the universe? How can the CBR look the same in all directions [16] when it comes from causally disconnected regions of space? This is referred to as the horizon problem. What was the source(s) of small and large scale structure formations? What is the origin of the small fluctuations in the CBR temperature? What happened before the Planck epoch? To answer some of these questions, inflation was constructed [17, 18]. Particle physics attempts to answer the remaining questions.

The particle physics standard model is based on the $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ gauge theory of the strong, weak and electromagnetic interactions. The non-abelian $\mathrm{SU}(3)_c$ part describes the strong interaction and the $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ part describes the electroweak interactions which combine the weak and electromagnetic interactions. At energies ~ 100 GeV, the standard model gauge group is spontaneously broken down to $\mathrm{SU}(3)_c \times \mathrm{U}(1)_Q$ by the vacuum expectation value of a single Higgs field, which is an SU(2) doublet. The electric charge Q is a linear combination of the weak isospin T_{3L} and of the weak hypercharge Y. The standard model contains 12 spin-1 gauge bosons: the 8 gluons of $\mathrm{SU}(3)_c$, the three $W_L^{\pm,0}$ of $\mathrm{SU}(2)_L$ and the $\mathrm{U}(1)_Y$ gauge boson (the $W_L^{\pm}$ bosons, the Z boson and the photon). There are three independent gauge coupling constants g_1, g_2 and g_3 associated with the three gauge groups $\mathrm{U}(1)_Y$, $\mathrm{SU}(2)_L$ and $\mathrm{SU}(3)_c$ respectively. The inverse of the 'structure constants', $\alpha_1^{-1} = \frac{4\pi}{g_1^2}$, $\alpha_2^{-1} = \frac{4\pi}{g_2^2}$ and $\alpha_3^{-1} = \frac{4\pi}{g_3^2}$, depend logarithmically on the energy scale. Interpolating their low energy values measured at LEP/SLC to very high energies, it is found that they roughly meet at $10^{14} - 10^{15}$ GeV with values $\alpha_1^{-1} = \alpha_2^{-1} = 43$ and $\alpha_3^{-1} \simeq 38$. In the minimal supersymmetric extension of the standard model, the three couplings all merge in a single point at a scale $\simeq 2 \times 10^{16}$ GeV with a value $\alpha_1^{-1} = \alpha_2^{-1} = 25 \simeq \alpha_3^{-1}$ [19].

Only left-handed fermions take part in weak interactions and hence only left-handed fermions transform non-trivially under $\mathrm{SU}(2)_L$. The fact that right-handed particles do not take part in weak interactions means that parity is maximally violated. The left-handed quarks and the left-handed leptons are grouped into, respectively, three SU(2) doublets, $\begin{pmatrix} u_L \\ d_L \end{pmatrix}$ and $\begin{pmatrix} e_L \\ \nu_L \end{pmatrix}$. The right-handed fields are SU(2) singlets, (e_R), (u_R) and (d_R). The standard model does not contain right-handed neutrino. Only quarks and gluons feel the strong interaction. Quarks form colour triplets, gluons colour octets, whereas leptons, other gauge bosons and the Higgs particle form colour singlets.

The standard model predicts the conservation of the colour and electric charges. It also predicts the conservation of baryon number B and lepton numbers L_e, L_μ and L_τ for each family. The standard model has been successfully tested at high energy colliders, with some predictions checked up to accuracy as high as 0.1% [22]. One predicted particle is however missing in these tables [22]: the electroweak Higgs boson.

The particle physics standard model faces problems and difficulties, as does the big-bang cosmology. It contains 18 free parameters which are not predicted. It does not predict the fermion masses nor the weak mixing angle. Why is the theory left-handed? Why is the electric charge quantised? Why do we observe three families? Why is the electroweak symmetry breaking scale ($\sim 10^2$ GeV) so small compared to the Planck scale ($\sim 10^{19}$ GeV)? Where does gravity fit in? The particle physics standard model also misses interesting features, such as neutrino masses, high-energy gauge coupling constant unification, and cold dark-matter candidates. To solve these problems, theories beyond the standard model have been constructed. These include grand unified theories (GUTs) and supersymmetry.

The basic idea of grand unification is to embed the standard model gauge group $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ into a larger simple group G which has a single gauge coupling constant g, so that the additional symmetries restrict the number of unpredicted parameters. GUTs predict the unification at high energies of the strong, weak and electromagnetic interactions into a single force. Any grand unified gauge group $\mathrm{G} \supset \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ should be consistent with low energy phenomena, and should be at least rank four. It should contain the 8 gluons, the two $W_L^\pm$ bosons, the Z boson and the photon; G will also contain extra gauge bosons, some of which can violate baryon and/or lepton number. As a consequence, GUTs predict proton decay. The particle representations of G should contain SU(2) doublets and SU(3) triplets. G should contain complex representations to which fermions will be assigned, so that low energy structures emerge. Extra scalar bosons in appropriate representations must be introduced in order to get an acceptable spontaneous symmetry breaking pattern from G down to the standard model gauge group. GUTs with rank greater or equal to five also predict extra fermions. One of them, transforming as a singlet under the standard model gauge group, can be identified with a right-handed neutrino. Such unified theories predict non-zero neutrino masses.

GUTs also provide the standard scenario for baryogenesis, via the out-of-equilibrium decay of heavy Higgs and gauge bosons which violate baryon and/or lepton number [5]. GUTs with heavy

Majorana right-handed neutrinos provide the standard scenario for leptogenesis [23, 24, 25].

The minimal GUT which unifies all matter, without introducing 'exotic' particles, except a right-handed neutrino, is SO(10) [26]. SO(10) has a sixteen dimensional spinorial representation to which all fermions, both left and right-handed (precisely, the left-handed and the charge conjugate of the right-handed ones or vice-versa), belonging to a single family, can be assigned. The sixteenth fermion transforms as a singlet under $SU(3)_c \times SU(2)_L \times U(1)_Y$ and hence can be identified with a right-handed neutrino. SO(10) predicts non zero neutrino masses. SO(10) contains 45 gauge bosons: the 8 gluons, the three $W_L^{\pm,0}$, the three $W_R^{\pm,0}$ of $SU(2)_R$ subgroup of SO(10), 30 gauge bosons which violate baryon number, and one gauge boson B' which violates $B - L$. The $U(1)_Y$ associated gauge boson is a linear combination of W_R^0 and B'. All the gauge bosons which are not contained in the standard model must acquire masses well above the electroweak symmetry breaking scale. GUTs such as E(6), E(7) or higher rank GUTs predict more fermions and even more gauge bosons. They contain many more parameters and so are more complicated than SO(10), but they do not give a better fit to the low energy data.

We should also mention that the minimal GUT, which is SU(5), is inconsistent with low energy data. Nonsupersymmetric SU(5) is ruled out because of the gauge coupling constant which do not all exactly meet, and supersymmetric SU(5) because it is inconsistent with proton lifetime measurements. It should be pointed out that in nonsupersymmetric grand unified theories with rank greater or equal to five, one (or more) intermediate symmetry breaking scale can be introduced; this scale can be chosen in such a way that the three coupling constants meet in a single point at $\sim 10^{15}$ GeV. Also, in SU(5), fermions must be assigned to different representations; so SU(5) does not unify matter, and neither does it predict right-handed neutrinos.

GUTs introduce new symmetries between different kinds of matter and between different kind of forces. Supersymmetry introduces a symmetry between fermions and bosons. It introduces new particles, new fermions and new bosons. Each particle of the nonsupersymmetric theory under consideration is associated with a particle with the same quantum numbers except from the spin; it differs from the spin of the nonsupersymmetric particle by half a unit. These are referred to as superparticles. Hence fermions with spin-$\frac{1}{2}$ are associated with spin-0 bosons, called sfermions, gauge bosons have spin-1 and are associated with spin-$\frac{1}{2}$ fermions, called gauginos, and Higgs bosons have spin-1 and are associated with spin-$\frac{1}{2}$ fermions, called Higgsinos. The minimal supersymmetric standard model (MSSM) is the minimal supersymmetric extension of the standard model. It contains the usual quarks, leptons and gauge bosons and their supersymmetric partners. It contains two Higgs doublets and their supersymmetric partners, one of which gives a mass to up-type quarks and one which gives a mass to d-type quarks. The gauge symmetry of the MSSM is still the $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge theory. For instance, the left-handed $SU(2)_L$ fermion doublet $\begin{pmatrix} e_L \\ \nu_L \end{pmatrix}$ is partnered with the scalar doublet $\begin{pmatrix} \tilde{e}_L \\ \tilde{\nu}_L \end{pmatrix}$. The Feynman rules for the supersymmetric partners allow the same interactions, although one must take into account the different spins of the particles.

An important assumption in the MSSM is the imposition of a discrete symmetry, known as R-parity, which takes values +1 for particles and −1 for superparticles. The assumption of unbroken R-parity restricts the possible interactions in the theory. It prevents rapid proton decay and stabilises the lightest superparticle, thus providing a cold dark-matter candidate. Some GUTs automatically conserve R-parity; this will be discussed in Sec. 1.3.

The study of supersymmetric GUTs is well motivated. Especially since, as was mentioned above, in the MSSM, the gauge coupling constants have been shown to merge in a single point at 2×10^{16} GeV. Further motivations are discussed in Sec. 1.3.

A consequence of grand unified theories, supersymmetric or not, is the formation of topological defects, according to the Kibble mechanism [27], at phase transitions associated with the spontaneous symmetry breaking of the grand unified gauge group G down to the standard model gauge group. Examples of such defects are monopoles, cosmic strings or domain walls. The Kibble mechanism relies on the fact that the Higgs field mediating the SSB of a group G down to a subgroup H of G must have taken different values in different regions of space which were not correlated. When the topology of the vacuum manifold $\frac{\mathrm{G}}{\mathrm{H}}$ is non trivial, topological defects then form. Monopoles form when the vacuum manifold contains non-contractible two-spheres, cosmic strings form when it contains non-contractible loops and domain walls form when it is disconnected. The topology of the vacuum manifold can be determined using homotopy theory. Hence, topological defects forming when G breaks down to H, can be classified in terms of the homotopy groups of the vacuum manifold $\frac{\mathrm{G}}{\mathrm{H}}$ [27]. If the fundamental homotopy group $\pi_0(\frac{\mathrm{G}}{\mathrm{H}}) \neq I$ is non trivial, domain walls form when G breaks down to H. If the first homotopy group $\pi_1(\frac{\mathrm{G}}{\mathrm{H}}) \neq I$ is non trivial, topological cosmic strings form. If the second homotopy group $\pi_2(\frac{\mathrm{G}}{\mathrm{H}}) \neq I$ is non trivial, monopoles form. If the subgroup H of G breaks later to a subgroup K of H, the defects formed when G breaks to H can either be stable or rapidly decay. The stability conditions are as follows. If the fundamental homotopy group $\pi_0(\frac{\mathrm{G}}{\mathrm{K}})$ is non trivial, the walls are topologically stable, if $\pi_1(\frac{\mathrm{G}}{\mathrm{K}})$ is non trivial, the strings are topologically stable and if $\pi_2(\frac{\mathrm{G}}{\mathrm{K}})$ is non trivial, the monopoles are topologically stable down to K.

As mentioned above, we are especially interested in SO(10) GUT. Consider then the phase transition associated with the breaking of SO(10) down to a subgroup G of SO(10), and apply the above results to this particular case; it will be useful later on. Note first that when we write SO(10) we mean its universal covering group Spin(10) which is simply connected. Since Spin(10) is connected and simply connected we have $\pi_2(\frac{\mathrm{SO(10)}}{\mathrm{G}}) = \pi_1(\mathrm{G})$ and $\pi_1(\frac{\mathrm{SO(10)}}{\mathrm{G}}) = \pi_0(\mathrm{G})$ and therefore the formation of monopoles and strings during the grand unified phase transition is governed by the non triviality of $\pi_1(\mathrm{G})$ and $\pi_0(\mathrm{G})$ respectively. If G breaks down later to a subgroup K of G, monopoles formed during the first phase transition will remain topologically stable after the second phase transition if $\pi_1(\mathrm{K}) \neq I$. Strings formed during the first phase transition will be topologically stable after the next phase transition if $\pi_0(\mathrm{K}) \neq I$.

Domain walls, because they are too heavy, and monopoles, because they are too abundant according to the Kibble mechanism, if present today, would dominate the energy density of the universe and hence are in conflict with the standard cosmology [7]. On the other hand, cosmic strings may have interesting cosmological consequences. They may help explain the formation of large scale structure, the baryon asymmetry of the universe, temperature fluctuations in the CBR [7] and could be part of the missing matter [28]. The formation of topological defects is unavoidable.

A solution to the monopole and domain wall problems is inflation [5, 17, 18]. The basic idea of inflation is that there was an epoch in the very early universe when the potential, vacuum energy density dominated the energy density of the universe, so that the cosmic scale factor grew exponentially. Regions initially within the causal horizon were then expanded to sizes much greater than the present Hubble radius.

In the following chapters, we will be mainly interested in hybrid inflation which was first introduced by Linde [20]. Hybrid inflationary scenarios use two scalar fields, $\mathcal{S}$ and ϕ. One, $\mathcal{S}$, is utilised to trap the second, ϕ, in a false vacuum state. Inflation then takes place as the field $\mathcal{S}$ slowly moves. When the field $\mathcal{S}$ reaches its critical value, the ϕ field quickly changes, reducing its energy density, and inflation ends. Initial conditions in hybrid inflationary scenario are 'chaotic', thermal equilibrium is not assumed, and thus hybrid inflation belongs to the general class of chaotic inflation models [21]. We will be interested in models where the $\mathcal{S}$ field is a scalar singlet and the ϕ field is a Higgs used to lower the rank of the group under consideration by one unit.

Inflation can solve the monopole problem, introduced by grand unified theories, and the big-bang horizon and flatness problems. Also, quantum fluctuations in the fields driving inflation may have been at the origin of the density perturbations that caused structure formation and temperature fluctuations in the CBR. Hence inflation is very promising.

Unfortunately, inflation usually requires very fine tuning in the parameters; the scalar field which drives inflation must have a very small self-coupling and the potential must be almost flat. The aim is therefore to find a theory in which such a scalar field with flat potential naturally emerges, so that inflation occurs 'naturally'. This goal seems to have been attained in supersymmetric grand unified models [29, 30, 31, 32, 33].

If inflation ends before the last spontaneous symmetry breaking down to the standard model gauge group has taken place, topological defects, such as cosmic strings, may still be present today. The formation of topological defects at the end of inflation has been studied in Refs. [31, 34]. Models in which monopoles or domain walls form at the end of inflation are cosmologically unacceptable. On the other hand, if cosmic strings form at the end of inflation, there may be important cosmological consequences. Recall that cosmic strings are good candidates for structure formation and that they may be necessary to explain the spectrum of temperature fluctuations in the CBR and the baryon asymmetry of the universe. The formation of cosmic strings at the end of inflation may be necessary. Hence, a unified model with both inflation, which inflates away all unwanted defects, and cosmic strings, forming at the end of inflation, is quite attractive. However, the construction of such a model is not easy.

In this thesis, we investigate some interesting features of cosmic strings. We look at the possibility that cosmic strings arising in a specific GUT model may catalyse proton decay and also that cosmic strings with right-handed neutrinos trapped as transverse zero modes in their core may be candidates for baryogenesis. We are especially interested in groups with rank greater or equal to five for the unified models, with particular interest in SO(10) GUT. We construct a grand unified model which contains both inflation and cosmic strings, as well as hot and cold dark-matter candidates and provides a mechanism for baryogenesis.

Several years ago, Callan and Rubakov suggested that grand unified monopoles could catalyse proton decay with a strong interaction cross-section [35]. This is known as the Callan-Rubakov effect. More recently, Perkins *et al.*[36] showed that the same effect could occur with cosmic strings; they were using a toy model. In Chap. 2, we investigate if this effect occurs when a specific GUT model is used. The model we use is a nonsupersymmetric SO(10) model with SO(10) broken down to the standard model via an $\mathrm{SU}(5)\times Z_2$ symmetry. We study the scattering of fermions off the abelian string which forms when SO(10) is broken down to $\mathrm{SU}(5) \times Z_2$. The elastic scattering cross-sections and the inelastic scattering cross-sections due to the coupling of fermions to gauge fields which violate baryon number in the core of the strings are computed.

The aim of Chapters 3 and 4 is to construct a grand unified model consistent with observations. We require that the model undergoes a period of inflation which arises naturally from the theory, and that cosmic strings form at the end of inflation. This work was motivated by the work of Dvali *et al.*[29] which "outlined how supersymmetric models can lead to a successful inflationary scenario without involving small dimensionless couplings".

In Chap. 3, we give the motivations for choosing supersymmetric SO(10) as grand unified theory. We analyse the conditions for topological defect formation in supersymmetric theories. The formation of topological defects in all possible spontaneous symmetry breaking patterns from supersymmetric SO(10) down to the standard model gauge group are then studied. Assuming that the universe underwent a period of inflation, as will be described in Chap. 4, and by requiring that the model be consistent with proton lifetime measurements, we select the only patterns consistent with observations.

In Chap. 4, we describe an inflationary scenario which arises naturally from the theory in supersymmetric SO(10) models. We give the form of the general superpotential which gives rise to an inflationary period and implements the spontaneous symmetry breaking pattern. We give a brief description of the evolution of the fields. We then consider one of the model selected in Chap. 3. We argued above that a model with both inflation and cosmic strings was a good framework for describing the very early universe. We therefore choose such a model. We construct the full superpotential. We study the evolution of the fields and the formation of topological defects before and at the end of inflation. Monopoles are inflated away and cosmic strings form at the end of inflation. The properties of the strings are presented. The dark-matter components of the model are analysed. The properties of a mixed scenario of cosmic string and inflationary large-scale structure formation are briefly discussed.

Motivated by baryogenesis via leptogenesis scenarios [23, 24, 25] introduced a decade ago [23], we present a new mechanism for leptogenesis in Chap. 5. The basic idea is that the out-of-equilibrium decay of heavy Majorana right-handed neutrinos released by decaying cosmic string loops produces a lepton asymmetry which is then converted into a baryon asymmetry. Theories in which such a scenario does work are identified.

We finally conclude in Chap. 6, summarising the results of the works carried out in the previous Chapters. We also give some ideas for future research works.

1.2 Massive neutrinos

1.2.1 Introduction

Grand unified theories with rank greater or equal to five predict extra fermions in addition to the usual quarks and leptons. One of them (one per fermion generation), a singlet under the standard model gauge group, can be interpreted as a right-handed neutrino. As a consequence, the observed neutrinos acquire a non-zero mass. Right-handed neutrinos are predicted to be massive Majorana particles. For a review of massive neutrinos, the reader is referred to Refs. [37, 38].

The existence of non-zero neutrino mass may have important cosmological and astrophysical implications [12]. In particular, if neutrinos are massive, they will account for the invisible matter of the universe. Recent analytical work on large scale structure formation strongly favours cosmological models with both hot and cold dark matter [39]. Massive neutrinos are the natural candidate for the hot dark matter component.

Non-zero neutrino masses also lead to neutrino flavour oscillations. These can explain, via the MSW mechanism [40], the apparent deficit in the flux of solar neutrinos as measured by the Homestake, Kamiokande, GALLEX and SAGE experiments. They can also explain the small ratio of muon to electron atmospheric neutrinos. Nevertheless, we should point out that the solar neutrino problem may still be a terrestrial or an astrophysical problem [41]. Only future experiments such as SNO, Superkamiokande, BOREXINO and HELLAZ, will be able to supply definitive proof that the solar neutrino problem is a consequence of physics beyond the electroweak standard model [41].

We conclude that there is strong theoretical support for neutrinos to be massive and probably also soon direct experimental support.

The experimental bounds on the neutrino masses, which may be Dirac or Majorana type, are as follows [22]

$$m_{\nu_e} \leq 15\mathrm{eV} \tag{1.7}$$
$$m_{\nu_\mu} \leq 170\mathrm{keV} \tag{1.8}$$
$$m_{\nu_\tau} \leq 24\mathrm{MeV}. \tag{1.9}$$

In the following chapters, we will be particularly interested in unified theories which contain an extra $\mathrm{U}(1)_{B-L}$ gauge symmetry, such as the simple U(1) extension of the standard model, $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1) \times \mathrm{U}(1)'$, left-right models or SO(10) and E(6) GUTs. In such theories, right-handed neutrinos acquire a heavy Majorana mass at the $B-L$ breaking scale.

Majorana particles are introduced in Sec. 1.2.2. In Sec. 1.2.3, we review the see-saw mechanism. In Sec. 1.2.4, we give the expansions of both Dirac and Majorana quantised fermion fields. The difference between Dirac and Majorana fermion fields is then obvious.

1.2.2 Majorana particles

Massive charged spin-$\frac{1}{2}$ particles can be described by four-component spinor fields Ψ which can be written as the sum of two two-component Weyl (or chiral) spinors,

$$\Psi = \Psi_L + \Psi_R = \frac{1+\gamma^5}{2}\Psi + \frac{1-\gamma^5}{2}\Psi = P_L\Psi + P_R\Psi \tag{1.10}$$

where, in the case that the mass of the particle can be ignored, Ψ_L (Ψ_R) annihilates a left (right)-handed particle or creates a right (left)-handed particle.

Under charge conjugation, the fields Ψ and $\overline{\Psi} = \Psi^\dagger\gamma^0$ transform as follows,

$$\Psi \stackrel{C}{\to} \Psi^c \equiv C\overline{\Psi}^T = C\gamma^0\Psi^* \tag{1.11}$$

$$\overline{\Psi} \stackrel{C}{\to} \overline{\Psi}^c \equiv -\Psi^T C^{-1} \tag{1.12}$$

where C is the charge conjugation matrix defined by $C^{-1}\gamma_\mu C = -\gamma_\mu^T$ and satisfies $C = -C^{-1} = -C^\dagger = -C^T$. Also we have

$$\Psi_{L,R} \stackrel{C}{\to} \Psi^c_{L,R} \equiv P_{L,R}\Psi^c = C\overline{\Psi}^T_{R,L} = C\gamma_0^T\Psi^*_{R,L} \tag{1.13}$$

$$\overline{\Psi}_{L,R} \stackrel{C}{\to} \overline{\Psi}^c_{L,R} \equiv -\Psi^c_{R,L}C^{-1}, \tag{1.14}$$

hence it follows that

$$\Psi_{L,R} = C\overline{\Psi}^{cT}_{R,L} \tag{1.15}$$

$$\overline{\Psi}_{L,R} = -\Psi^{cT}_{R,L}C^{-1}. \tag{1.16}$$

A Majorana spinor Ψ_M is by definition a spinor which is proportional to its own charge conjugate,

$$\Psi^c_M = \pm\Psi_M \tag{1.17}$$

and thus has only two independent components.

In the zero-mass limit the relation $\Psi^c_M = \Psi_M$ holds. Obviously, a Majorana spinor can only describe non-electrically-charged spin-$\frac{1}{2}$ particles. We also see from (1.17) the anti-particle of Majorana particle is the particle itself.

1.2.3 Majorana mass terms and the see-saw mechanism

Recall that the left and right-handed components of a Dirac spinor fields Ψ_L and Ψ_R are independent, and are usually used to describe the left and right-handed fermions as in the standard model. But one can as well choose Ψ_L and Ψ^c_L, or Ψ^c_R and Ψ_R as independent fields to describe left and right-handed particles. This is usually the case in grand unified theories where left and right-handed particles are assigned to the same multiplet (since gauge interactions conserve chirality, left and right-handed fields cannot be assigned to the same multiplet, see App. A).

However Ψ_L and Ψ_R^c or Ψ_R and Ψ_L^c are not independent. They are related by Eq. (1.13). Using Ψ_L and Ψ_R^c or Ψ_R and Ψ_L^c, one can construct two Majorana spinors

$$\Psi_{M_L} = \Psi_L + \Psi_R^c \quad \text{and} \quad \Psi_{M_R} = \Psi_R + \Psi_L^c. \tag{1.18}$$

Majorana mass terms can be constructed for both Ψ_{M_L} and Ψ_{M_R}

$$M_L \overline{\Psi_{M_L}} \Psi_{M_L} = M_L(\overline{\Psi}_L \Psi_R^c + h.c) \tag{1.19}$$
$$M_R \overline{\Psi_{M_R}} \Psi_{M_R} = M_R(\overline{\Psi}_R \Psi_L^c + h.c). \tag{1.20}$$

These Majorana mass terms violate lepton number by two units, and hence may lead to interesting lepton number violating processes in the early universe (see Ref. [23] and chapter 4).

Therefore the general Lagrangian for free neutrino fields can be written as

$$\begin{aligned} L &= \overline{\nu_L}\gamma^\mu\partial_\mu\nu_L + \overline{\nu_R}\gamma^\mu\partial_\mu\nu_R + M_D(\overline{\nu_L}\nu_R + \overline{\nu_R}\nu_L) \\ &\quad + \frac{M_L}{2}(\overline{\nu_R^c}\nu_L + h.c) + \frac{M_R}{2}(\overline{\nu_L^c}\nu_R + h.c) \end{aligned} \tag{1.21}$$

where M_D is the neutrino Dirac mass and M_L and M_R are the Majorana masses of the left-handed and right-handed neutrino fields respectively.

In order to understand the physical content of this Lagrangian, we introduce new fields f and F

$$f = \frac{\nu_L + \nu_R^c}{\sqrt{2}} \quad F = \frac{\nu_R + \nu_L^c}{\sqrt{2}} \tag{1.22}$$

and the Lagrangian given in Eq. (1.21) is equivalent to

$$L = \overline{f}\gamma^\mu\partial_\mu f + \overline{F}\gamma^\mu\partial_\mu F + (\overline{f}, \overline{F}) \begin{pmatrix} M_L & M_D \\ M_D & M_R \end{pmatrix} \begin{pmatrix} f \\ F \end{pmatrix} \tag{1.23}$$

and the matrix

$$M = \begin{pmatrix} M_L & M_D \\ M_D & M_R \end{pmatrix} \tag{1.24}$$

is called the neutrino mass matrix.

In the models of interest, which contain a $U(1)_{B-L}$ symmetry and where the see-saw mechanism can be implemented [42], the left-handed neutrino Majorana mass vanishes, $M_L = 0$. The right-handed neutrino Majorana mass M_R is generated when the rank of the group is reduced by one unit, as a consequence of $B - L$ breaking. It comes from Yukawa coupling of the Majorana field describing the right-handed neutrino to the Higgs field ϕ_{B-L} used to break $B - L$, $\lambda < \Phi_{B-L} > \overline{\nu}_L^c \nu_R + h.c.$. Hence M_R is expected to be the order of the $B - L$ breaking scale, but it also depends on the value of the Yukawa coupling constant λ. Therefore, even in supersymmetric models where the $B-L$ symmetry breaking scale can be as high as 10^{16} GeV, we can still get right-handed neutrino Majorana masses in the observational bounds for neutrinos masses by assuming very small Yukawa coupling constant. Indeed, this is fine tuning. For nonsupersymmetric theories however, η_{B-L} usually lies in the range $10^{12}-10^{13}$ GeV, and no fine tuning is required. The Dirac neutrino mass is generated by the electroweak Higgs doublet, and hence is expected to be the order of the masses of the related charged up-type quarks (e.g. SO(10)). The neutrino mass matrix then becomes

$$M = \begin{pmatrix} 0 & M_D \\ M_D & M_R \end{pmatrix} \tag{1.25}$$

with $M_R \geq M_D$.

In order to find the physical fields and masses, we must diagonalise M. We find that the eigenvalues of the neutrino mass matrix are

$$\begin{aligned} M_N &\simeq M_R & (1.26) \\ \text{and} \quad M_\nu &\simeq -\frac{M_D^2}{M_R}, & (1.27) \end{aligned}$$

and the corresponding eigenvector fields are

$$\begin{aligned} N &\simeq F + \frac{M_D}{M_R} f & (1.28) \\ \nu' &\simeq f - \frac{M_D}{M_R} F. & (1.29) \end{aligned}$$

In order to get a positive mass for the light neutrino, we take the physical neutrino field to be

$$\nu = \gamma^5 \nu'. \tag{1.30}$$

In terms of ν and N, the Lagrangian (1.24) becomes

$$L = \overline{\nu}\gamma^\mu \partial_\mu \nu + \overline{N}\gamma^\mu \partial_\mu N + M_\nu \overline{\nu}\nu + M_N \overline{N}N \tag{1.31}$$

with

$$M_\nu = \frac{M_D^2}{M_R} \quad \text{and} \quad M_N = M_R. \tag{1.32}$$

Since we expect M_D to be the order of the associated charged lepton or quark, $M_D \simeq M_{q\text{or}l}$, we see that

$$M_\nu . M_N = M_{q,l}^2. \tag{1.33}$$

This is the famous see-saw relation [42].

The light and heavy neutrinos ν' and N are Majorana particles.

1.2.4 Quantised fermions fields

Dirac Case

Using the normalisation

$$\overline{u}^{\alpha}(k)u(k)_{\alpha} = \delta_{\alpha\alpha'}\,. \tag{1.34}$$

the Dirac fermion field operator has the form [6]

$$\Psi(x) = \int \frac{d^3k}{(2\pi)^3}\frac{m}{k_0}\sum_{\alpha=1,2}\Big[b_{\alpha}(k)u^{\alpha}(k)e^{-ik.x} + d^{\dagger}_{\alpha}(k)v^{\alpha}(k)e^{ik.x}\Big] \tag{1.35}$$

$$\overline{\Psi}(x) = \int \frac{d^3k}{(2\pi)^3}\frac{m}{k_0}\sum_{\alpha=1,2}\Big[b^{\dagger}_{\alpha}(k)\overline{u}^{\alpha}(k)e^{ik.x} + d_{\alpha}(k)\overline{v}^{\alpha}(k)e^{-ik.x}\Big] \tag{1.36}$$

where $u(k)$ and $v(k)$ are positive and negative energy spinors solutions of the Dirac equation. $b^{\dagger}_{\alpha}(k)$ and $b_{\alpha}(k)$, $d^{\dagger}_{\alpha}(k)$ and $d_{\alpha}(k)$ are, respectively, the creation and annihilation operators of a particle and an antiparticle with four-momentum k and helicity α.

The operators $b_{\alpha}(k)$ and $b^{\dagger}_{\alpha}(k)$, $d_{\alpha}(k)$ and $d^{\dagger}_{\alpha}(k)$ satisfy the anti-commutation relations

$$\{b_{\alpha}(k), b^{\dagger}_{\alpha'}(k')\} = \{d_{\alpha}(k), d^{\dagger}_{\alpha'}(k')\} = (2\pi)^3\frac{k_0}{m}\delta^3(\mathbf{k}-\mathbf{k}')\delta_{\alpha\alpha'} \tag{1.37}$$

$$\{b_{\alpha}(k), b_{\alpha'}(k')\} = \{b^{\dagger}_{\alpha}(k), b^{\dagger}_{\alpha'}(k')\} = 0 \tag{1.38}$$

$$\{d_{\alpha}(k), d_{\alpha'}(k')\} = \{d^{\dagger}_{\alpha}(k), d^{\dagger}_{\alpha'}(k')\} = 0\,. \tag{1.39}$$

Majorana case

The Majorana field operator has the form [37, 38]

$$\Psi(x) = \int \frac{d^3k}{(2\pi)^3}\frac{m}{k_0}\sum_{\alpha=1,2}\Big[b_{\alpha}(k)u^{\alpha}(k)e^{-ik.x} + \lambda b^{\dagger}_{\alpha}(k)v^{\alpha}(k)e^{ik.x}\Big] \tag{1.40}$$

$$\overline{\Psi}(x) = \int \frac{d^3k}{(2\pi)^3}\frac{m}{k_0}\sum_{\alpha=1,2}\Big[b^{\dagger}_{\alpha}(k)\overline{u}^{\alpha}(k)e^{ik.x} + \lambda b_{\alpha}(k)\overline{v}^{\alpha}(k)e^{-ik.x}\Big] \tag{1.41}$$

where λ is an arbitrary phase factor which is called the creation phase factor [43]. $u^{\alpha}(k)$ and $v^{\alpha}(k)$ are positive and negative energy Dirac spinors; they satisfy the charge conjugation relation

$$v^{\alpha}(k) = C\Big[\overline{u}^{\alpha}(k)\Big]^T = u^{\alpha}(-k). \tag{1.42}$$

The operators $b_{\alpha}(k)$ and $b^{\dagger}_{\alpha}(k)$ satisfy the canonical anti-commutation relations

$$\{b_{\alpha}(k), b^{\dagger}_{\alpha'}(k')\} = (2\pi)^3\frac{k_0}{m}\delta^3(\mathbf{k}-\mathbf{k}')\delta_{\alpha\alpha'} \tag{1.43}$$

$$\{b_{\alpha}(k), b_{\alpha'}(k')\} = \{b^{\dagger}_{\alpha}(k), b^{\dagger}_{\alpha'}(k')\} = 0 \tag{1.44}$$

and, respectively, are the creation and annihilation operators of a Majorana particle with four-momentum k and helicity α.

The only difference between Majorana and Dirac quantised fermion fields is that the creation and annihilation operators are different for the particle and the antiparticle in the Dirac case, whereas they are the same in the case of Majorana fermions.

1.3 Supersymmetry

1.3.1 Motivation

In the past few years, supersymmetric GUTs have received much attention. We give below the motivations for studying supersymmetric GUTs. In Sec. 1.3.2, we introduce the notion of superspace, superfields and superpotentials and explain how to construct a supersymmetric Lagrangian.

Recall that the main motivation for studying supersymmetric GUTs rather than nonsupersymmetric ones has to do with the running of the gauge coupling constants. In the minimal supersymmetric standard model (MSSM), with supersymmetry broken at $\sim 10^3$ GeV, the hypercharge, weak isospin and QCD couplings meet in a single point at 2×10^{16} GeV, with a value $\alpha^{-1} \simeq 25$ [19]. There is no need to have an intermediate symmetry breaking scale, but an intermediate symmetry breaking scale is not forbidden either. The second main motivation for supersymmetry is to solve the hierarchy problem, related to the fourteen orders of magnitude separating the GUT scale Higgs vacuum expectation value (VEV) from the electroweak scale Higgs VEV.

Supersymmetric GUTs also provide a cold dark matter candidate, the lightest superparticle (LSP) which is stable if R-parity remains unbroken at low energy. $R = (-1)^{3(B-L)+2S}$, where B and L are respectively baryonic and leptonic charges and S is the particle spin. So defined, R-parity has value $(+1)$ for all particles and value (-1) for all antiparticles. There are three candidates for the LSP : the lightest neutralino, the sneutrino and, if gravity is included, the gravitino. There are many arbitrary parameters in the MSSM, and it does not predict what the LSP is. The situation is different in supergravity models [13]. Note also that R-parity has to be kept unbroken at low energy in order to forbid rapid proton decay. R-parity has to be artificially imposed in the MSSM, whereas in grand-unified models which contain a $\mathrm{U}(1)_{B-L}$ gauge symmetry, such as SO(10), R is automatically conserved if all Higgs vacuum expectation values used to implement the symmetry breaking pattern carry $3(B-L)$ [44]. In that case, $\mathrm{U}(1)_{B-L}$ is broken down to a gauged Z_2 subgroup which acts as R-parity. In SO(10), safe representations, those which conserve R, are the 10, 45, 54, **126**, 210 dimensional representations. Unsafe representations, which break R, are the **16** and 144 dimensional representations.

The nature of dark-matter must have influenced the density perturbations which lead to galaxy and large scale structure formation. Recent analysis shows that mixed cold and hot dark-matter scenarios work well [39]. This strongly favours supersymmetric particle physics models with stable LSP and massive neutrinos.

There is not yet experimental evidence for supersymmetry. If supersymmetry is fundamental to nature, it has to be a broken symmetry. The mechanism for supersymmetry breaking remains a fundamental problem. Searches for supersymmetry are being (and will be) conducted at high energy colliders [45], in double beta decay experiments [46], in proton decay experiments [47] and in dark matter detection experiments [48].

1.3.2 Constructing supersymmetric Lagrangians

We give here a brief review of supersymmetry. We introduce the supersymmetry algebra, the notion of superspace, superfields and superpotentials. For further details on supersymmetry the reader is referred to Ref. [9]. We explain how to construct supersymmetric Lagrangians with superfields and how to derive the effective scalar potential from a given superpotential.

In addition to the usual Lorentz-Poincaré generators, the six generators $M^{\mu\nu}$ of the Lorentz group and the four generators P^μ of the translation group ($\mu, \nu = 0...3$), supersymmetry introduces two-component Weyl spinor generators Q^A_α ($\alpha = 1, 2,\ A = 1...N$) which can change the spin of a particle by half-a-unit. In $N = 1$ supersymmetry there is one such generator, and there are $A = N$ generators in N-extended supersymmetry. We will restrict ourselves to $N = 1$ supersymmetry.

The generator Q_α is a two-component left-handed Weyl spinor generator and its adjoint hermitian conjugate denoted $\overline{Q}_{\dot\alpha}$ is a two-component right-handed Weyl spinor. The $N = 1$ supersymmetry algebra is

$$\{Q_\alpha, Q_\beta\} = \{\overline{Q}_{\dot\alpha}, \overline{Q}_{\dot\beta}\} = 0 \tag{1.45}$$

$$\{Q_\alpha, \overline{Q}_{\dot\beta}\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu \tag{1.46}$$

$$[Q_\alpha, P_\mu] = [\overline{Q}_{\dot\alpha}, P_\mu] = 0 \tag{1.47}$$

$$[M^{\mu\nu}, Q_\alpha] = -i(\sigma^{\mu\nu})^\beta_\alpha Q_\beta \tag{1.48}$$

$$[M^{\mu\nu}, \overline{Q}^{\dot\alpha}] = -i(\overline{\sigma}^{\mu\nu})^{\dot\alpha}_{\dot\beta} \overline{Q}^{\dot\beta}\ . \tag{1.49}$$

The Weyl indices are lowered and raised with the help of the tensors $\epsilon_{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ and $\epsilon^{\alpha\beta} = (\epsilon_{\alpha\beta})^{-1}$.

The usual representation of this algebra involves *vector* and *chiral* supermultiplets. Each supermultiplet contains the same number of fermions and bosons. The *chiral* supermultiplets contain a chiral spin-$\frac{1}{2}$ fermion and a spin-0 scalar boson. The *vector* multiplets contain a spin-1 vector boson and a Majorana spin-$\frac{1}{2}$ fermion.

Although it is possible to construct Lagrangians directly from the component fields belonging to a supermultiplet, the introduction of the notion of superspace and the use of superfields defined on the superspace make things much easier. A function defined on the superspace depends on the usual spacetime coordinates x^μ and on new coordinates θ and $\overline{\theta}$ which transform as two-component Weyl spinors elements of a Grassman algebra $\{\theta_i, \theta_j\} = \{\overline{\theta}_i, \overline{\theta}_j\} = \{\theta_i, \overline{\theta}_j\} = 0$. A superfield $S(x^\mu, \theta, \overline{\theta})$ can be expanded as a power series in θ and $\overline{\theta}$. The coefficients of the various powers of θ and $\overline{\theta}$ are the 'component' fields, which are ordinary scalar, vector and Weyl spinor fields.

The action of the supersymmetry algebra on $S(x^\mu, \theta, \overline{\theta})$ is

$$\begin{aligned} S(x^\mu, \theta, \overline{\theta}) \quad &\rightarrow \quad \exp(i(\theta Q + \overline{\theta Q} - x^\mu P_\mu)) S(a^\mu, \alpha, \overline{\alpha}) \\ &= S(x^\mu + a^\mu - i\alpha\sigma_\mu\overline{\theta} + i\theta\sigma_\mu\overline{\alpha}, \theta + \alpha, \overline{\theta} + \overline{\alpha})\ , \end{aligned} \tag{1.50}$$

which is generated by

$$P_\mu = i\partial_\mu \tag{1.51}$$

$$iQ_\alpha = \frac{\partial}{\partial\theta^\alpha} - i\sigma^\mu_{\alpha\dot\alpha}\overline{\theta}^{\dot\alpha}\partial_\mu \tag{1.52}$$

$$i\overline{Q}_\alpha = -\frac{\partial}{\partial\overline{\theta}^{\dot\alpha}} + i\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\partial_\mu\ . \tag{1.53}$$

A superfield is not at first instance an irreducible representation of the supersymmetry algebra. To get irreducible representations, we must impose supersymmetric constraints. Chiral superfields Φ are subject to the condition that

$$(-\frac{\partial}{\partial\overline{\theta}^{\dot\alpha}} - i\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\partial_\mu)\Phi = 0. \tag{1.54}$$

The component fields of a chiral superfield form a chiral multiplet. The most general chiral superfield Φ solution of (1.54) is

$$\Phi(y^\mu, \theta) = \phi(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y) \tag{1.55}$$

where $y^\mu = x^\mu + i\theta\sigma^\mu\overline{\theta}$, ϕ and F are complex scalar fields and ψ is a left-handed Weyl spinor field. The field F is called an auxiliary field.

Vector superfields V satisfy the condition $V^\dagger = V$. The component fields of a vector superfield form the vector multiplet. In the Wess-Zumino gauge, a vector superfield can be written as

$$V(x,\theta,\overline{\theta}) = \theta\sigma^\mu\overline{\theta}V_\mu(x) + \theta\theta\overline{\theta\lambda}(x) + \overline{\theta\theta}\theta\lambda(x) + \frac{1}{2}\theta\theta\overline{\theta\theta}D(x) \tag{1.56}$$

where V_μ is a vector field, and λ and $\overline{\lambda}$ are Weyl spinor fields and D is a scalar field. D is called an auxiliary field.

The product of two or three chiral superfields is a chiral superfield. On the other hand, the product $\Phi^\dagger\Phi$ is not a chiral superfield. It is a vector superfield. The coefficient of $\theta\theta$ in the product of two or three superfields is called a F-term in analogy with (1.55), and the coefficient of $\overline{\theta\theta}\theta\theta$ in a vector superfield is called a D-term in analogous with (1.56). Under a supersymmetry transformation, the change in the F-terms as well as in the D-terms is a total divergence. The space-time integral $\int d^4x$ of a total divergence vanishes (provided that the fields are zero at infinity, which is always assumed). Hence one can construct supersymmetric Lagrangians in terms of F and D-terms. A supersymmetric Lagrangian has the general form

$$L = \sum_i [\Phi_i^\dagger\Phi_i] + ([W(\Phi)]_F + h.c.) \tag{1.57}$$

where $W(\Phi)$ is a function of the chiral superfields Φ_i consistent with the considered gauge symmetry and at most cubic in the fields (in order to be renormalisable).

Given a superpotential, the auxiliary fields may be calculated as follows,

$$F_{\phi_i} = \frac{\partial W}{\partial\phi_i} \tag{1.58}$$

$$D^\alpha = g\sum_{a,b}\overline{\phi}_a T_b^{\alpha a}\phi^b \tag{1.59}$$

where the ϕ_i are the scalar components of the chiral superfields Φ_i and T^α are the generators of the gauge group. The scalar potential may be calculated in terms of the F-terms in Eq. (1.58) and the D-terms in Eq. (1.59),

$$V(\phi_i) = \sum_i |F_{\phi_i}|^2 + \sum_\alpha |D^\alpha|^2. \tag{1.60}$$

The only cases where one must pay attention to the D-terms is when a U(1) symmetry is broken that is when the rank of the gauge group is lowered by one unit.

The energy of any non-vacuum supersymmetric state is always positive definite, and the vacuum state must have zero energy. If the vacuum energy is non-zero, supersymmetry is broken. It is easy to see from Eq. (1.60) that supersymmetry breaking can occur if a F-term or a D-term gets a non-vanishing expectation value (VEV). If we want to keep supersymmetry unbroken, we must ensure that the F-terms and D-terms get a vanishing VEV.

These methods will be applied in Chap. 4, where a specific supersymmetric grand unified model is built.

Chapter 2

Scattering off an SO(10) cosmic string

2.1 Introduction

Modern particle physics and the hot big-bang model suggest that the universe underwent a series of phase transitions at early times at which the underlying symmetry changed. At such phase transitions topological defects [7] could be formed. Such topological defects, in particular cosmic strings, would still be around today and provide a window into the physics of the early universe. In particular, cosmic strings arising from a grand unified phase transition are good candidates for the generation of density perturbations in the early universe which lead to the formation of large scale structure [7]. They could also give rise to the observed anisotropy in the microwave background radiation [7].

Cosmic strings also have interesting microphysical properties. Like monopoles [35], they can catalyse baryon violating processes [36]. This is because the full grand unified symmetry is restored in the core of the string, and hence grand unified, baryon violating processes are unsuppressed. In [36] it was shown that the cosmic string catalysis cross-section could be a strong interaction cross-section, independent of the grand unified scale, depending on the flux on the string. Unlike the case of monopoles, where there is a Dirac quantisation condition, the string cross-section is highly sensitive to the flux, and is a purely quantum phenomena. Defect catalysis is potentially important since catalysis cross-sections have been shown to be the order of the strong interaction [35, 36]. It has already been used to bound the monopole flux [49], and could erase a primordial baryon asymmetry [50]. It is, thus, important to calculate the string catalysis cross-section in a realistic grand unified theory. In [36] a toy model based on a U(1) theory was used. In a grand unified theory the string flux is given by the gauge group, and cannot be tuned.

A cosmic string is essentially a flux tube. Hence the elastic cross-section [51] is just an Aharonov-Bohm cross-section [52], depending on the string flux. This gives the dominant energy loss in a friction dominated universe [53]. Since the string flux is fixed for any given particle species it is important to check that the Aharonov-Bohm cross-section persists in a realistic grand unified theory.

In this chapter, we study the scattering of fermion off the abelian string arising in the breaking of SO(10) down to $\mathrm{SU}(5)\times Z_2$ [54]. $\mathrm{SU}(5)\times Z_2$ is then broken down to $\mathrm{SU}(3)_c\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y\times Z_2$. These strings have been studied elsewhere [55, 56]. We calculate the elastic cross-sections and the baryon number violating cross-sections due to the coupling to gauge fields in the core of the string, by both a first quantised method and a perturbative second quantised method.

In Sec. 2.2 we define an SO(10) string model. We give 'top-hat' forms for the Higgs and gauge fields forming the string, since the 'top-hat' core model does not affect the cross-sections of interest [36]. Looking at the microscopic structure of the string core, we introduce the baryon number violating gauge fields of SO(10) present in the core of the string.

In Sec. 2.3.1 we review the method used to calculate the scattering cross-sections. There are two different approaches. A fundamental quantum mechanical one and a perturbative second quantised method [57, 36]. The latter consists in calculating the geometrical cross-section, i.e. the scattering cross-section for free fermionic fields. The catalysis cross-section is then enhanced by an amplification factor to the power of four.

In Sec. 2.3.2 we derive the equations of motion. In order to simplify the calculations and to get a fuller result, we also consider a 'top-hat' core model for the gauge fields mediating quark to lepton transitions.

In Sec. 2.3.3 and Sec. 2.3.4 we calculate the solutions to the equations of motion outside and inside the string core respectively, and in Sec. 2.3.5 we match our solutions at the string radius. In Sec. 2.3.6 we calculate the scattering amplitude for incoming plane waves of linear combinations of the quark and electron fields.

We use these results in Sec. 2.4 and Sec. 2.5 in order to calculate the scattering cross-sections of incoming beams of pure single fermion fields. In Sec. 2.4 we calculate the elastic cross-sections. And in Sec. 2.7 we calculate the baryon number violating cross-sections.

In Sec. 2.6 we derive the catalysis cross-section using the second quantised method of Refs. [57, 36]. The second-quantised cross-sections are found to agree with the first quantised cross-section of Sec. 2.5.

In Appendix A we give a brief review on SO(10) theory, and give an explicit notation used everywhere in this chapter. Appendices B.1 and B.2 contain the technical details of the external and internal solutions calculations. Finally, Appendix B.3 is a discussion of the matching conditions at the core radius.

2.2 An SO(10) string

In this section, we describe the abelian cosmic string which arises during the phase transition associated with the spontaneous symmetry breaking of SO(10) down to SU(5) $\times Z_2$. We define the set of Higgs fields used to implement the symmetry breaking pattern from SO(10) down to low energy. We then model our string using a 'top-hat' core model [36] and study the baryon number violating gauge interactions which occur inside the core of the string.

Kibble *et al.*[54] first pointed out that cosmic strings form when SO(10) breaks down to SU(5) $\times Z_2$. Following Ref. [54], we then assume that SO(10) is broken down to SU(5) $\times Z_2$ via the vacuum expectation value (VEV) of a Higgs field in the **126** dimensional representation of SO(10), which we call ϕ_{126}. SU(5) $\times Z_2$ is then broken down to the standard model gauge group with added Z_2 discrete symmetry, subgroup of the Z_4 centre of SO(10), with the VEV of a Higgs field in the 45 dimensional representation, which we call A_{45}. The standard model gauge group is finally broken down to $\mathrm{SU}(3)_c \times \mathrm{U}(1)_Q \times Z_2$ by the VEV of a Higgs field in the 10 dimensional representation of SO(10), H_{10}. The symmetry breaking pattern is therefore as follows,

$$\mathrm{SO}(10) \stackrel{<\phi_{126}>}{\rightarrow} \mathrm{SU}(5) \times Z_2 \stackrel{<A_{45}>}{\rightarrow} \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times Z_2 \stackrel{<H_{10}>}{\rightarrow} \mathrm{SU}(3)_c \times \mathrm{U}(1)_Q \times Z_2$$

The components of the 10-multiplet which acquire a VEV correspond to the usual Higgs doublet. The first transition is achieved by giving vacuum expectation value to the component of the **126** which transforms as a singlet under SU(5).

The first homotopy group $\pi_1(\mathrm{SO}(10)/\mathrm{SU}(5) \times Z_2) = \pi_0(\mathrm{SU}(5) \times Z_2 = Z_2$, and therefore Z_2 strings are formed when SO(10) breaks down to SU(5) $\times Z_2$. Furthermore, since $\pi_0(\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times Z_2) = \pi_0(\mathrm{SU}(3)_c \times \mathrm{U}(1)_Q \times Z_2 = Z_2$, the strings are topologically stable down to low energy.

In terms of SU(5), the 45 generators of SO(10) can be decomposed as follows,

$$45 = 24 + 1 + 10 + \bar{10} \tag{2.1}$$

From the 45 generators of SO(10), 24 belong to SU(5), 1 generator corresponds to the $\mathrm{U}(1)'$ symmetry in SO(10) not embedded in SU(5) and there are 20 remaining ones. Therefore the breaking of SO(10) to SU(5) $\times Z_2$ induces the creation of two types of strings. An Abelian one, corresponding to the $\mathrm{U}(1)'$ symmetry, and a non abelian one made with linear combinations of the 20 remaining generators. In this paper we are interested in the abelian strings since the non abelian version are Alice strings, and would result in global quantum number being ill-defined, and hence unobservable [58]. We note that there is a wide range of parameters where the non abelian strings have lower energy [56]. However, since the abelian string is topologically stable, there is a finite probability that it could be formed by the Kibble mechanism [27].

In the appendix A, we give a brief review of SO(10) [26]. With that notation, the Lagrangian is

$$L = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \Phi_{126})^\dagger (D^\mu \Phi_{126}) - V(\Phi) + L_F \tag{2.2}$$

where $F_{\mu\nu} = -i\, F^a_{\mu\nu}\tau_a,\ \tau_a\ a = 1, ..., 45$ are the 45 generators of SO(10). Φ_{126} is the Higgs **126**, the self-dual anti-symmetric 5-index tensor of SO(10). L_F is the fermionic part of the Lagrangian. In the covariant derivative $D_\mu = \partial_\mu + ieA_\mu$, $A_\mu = A^a_\mu \tau_a$ where A^a_μ a = 1,...,45 are 45 gauge fields of SO(10).

If we call τ_{str} the generator of the abelian string, τ_{str} will be given by the diagonal generators of SO(10) not lying in SU(5) that is,

$$\tau_{str} = \frac{1}{5}\left(M_{12} + M_{34} + M_{56} + M_{78} + M_{9\,10}\right) \tag{2.3}$$

where $M_{ij} : i, j = 1...10$ are the 45 SO(10) generators defined in appendix A in terms of the generalised gamma matrices. Numerically, this gives,

$$\tau_{str} = diag(\frac{1}{2}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{-3}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{-3}{10}, \frac{1}{10}, \frac{-3}{10}, \frac{-3}{10}, \frac{-3}{10})\,. \tag{2.4}$$

The results of Perkins *et al.*[36] find that the greatest enhancement of the cross-section is for fermionic charges close to integer values. Thus, from equation (2.4), we expect no great enhancement; the most being due to the right-handed neutrino.

We are going to model our string as is usually done for an abelian U(1) string [7]. That is, we take the string along the z axis, resulting in the Higgs Φ_{126} and the gauge fields A_μ of the string to be independent of the z coordinate, depending only on the polar coordinates (r, θ). Here A_μ is the gauge field of the string, obtained from the product $A_\mu = A_{\mu,str}\tau_{str}$. The solution for the abelian string can be written as,

$$\begin{aligned} \Phi_{126} &= f(r)\, e^{i\tau_{str}\theta}\, \Phi_0 = f(r)\, e^{i\theta}\, \Phi_0 && (2.5) \\ A_\theta &= -\frac{g(r)}{er}\tau_{str} && \\ A_r &= A_z = 0 && (2.6) \end{aligned}$$

where Φ_0 is the vacuum expectation value of the Higgs **126** in the 1_{10} direction. The functions $f(r)$ and $g(r)$ describing the behaviour the Higgs and gauge fields forming the string are approximately given by

$$f(r) = \begin{cases} \eta & r \geq R \\ (\frac{\eta r}{R}) & r < R \end{cases} \quad , \quad g(r) = \begin{cases} 1 & r \geq R \\ (\frac{r}{R})^2 & r < R \end{cases} \tag{2.7}$$

where R is the radius of the string. $R \sim \eta^{-1}$, where η is the grand unified scale, assumed to be $\eta \sim 10^{15}$ GeV. In order to simplify the calculations and to get a fuller result we use the top-hat core model, since it has been shown not to affect the cross-sections of interest. The top-hat core model assumes that the Higgs and gauge fields forming the string are zero inside the string core. Hence, $f(r)$ and $g(r)$ are now given by,

$$f(r) = \begin{cases} \eta & r \geq R \\ 0 & r < R \end{cases} \quad , \quad g(r) = \begin{cases} 1 & r \geq R \\ 0 & r < R \end{cases} \,. \tag{2.8}$$

The full SO(10) symmetry is restored in the core of the string. SO(10) contains 30 gauge bosons leading to baryon decay. These are the bosons X and Y, and their conjugates, of SU(5) plus 18 other gauge bosons usual called X', Y' and X_s, and their conjugates. Therefore inside the string core, there are quark to lepton transitions mediated by the gauge bosons X, X', Y, Y' and X_s and we expect the string to catalyse baryon number violating processes in the early universe.

The X, X', Y, Y' and X_s gauge bosons are associated with non diagonal generators of SO(10). For the electron family, the relevant part of the Lagrangian is given by,

$$L_x = \bar{\Psi}_{16}\,(ie\gamma^\mu(X_\mu\tau^X + X'_\mu\tau^{X'} + Y_\mu\tau^Y + Y'_\mu\tau^{Y'} + X_{s\mu}\tau^{X_s}))\,\Psi_{16} \tag{2.9}$$

where τ^X, $\tau^{X'}$, τ^Y, and $\tau^{Y'}$ and τ^{X_s} are the non diagonal generators of SO(10) associated with the X, X', Y, Y' and X_s gauge bosons respectively.

Expanding equation (2.9) gives [60],

$$\begin{aligned} L_x \;=\; & \frac{g}{\sqrt{2}}X^\alpha_\mu[-\epsilon_{\alpha\beta\gamma}\bar{u}^{c\gamma}_L\gamma^\mu u^\beta_L + \bar{d}_{L\alpha}\gamma^\mu e^+_L + \bar{d}_{R\alpha}\gamma^\mu e^+_R] \\ & + \frac{g}{\sqrt{2}}Y^\alpha_\mu[-\epsilon_{\alpha\beta\gamma}\bar{u}^{c\gamma}_L\gamma^\mu d^\beta_L - \bar{d}_{R\alpha}\gamma^\mu\nu^c_e - \bar{u}_{L\alpha}\gamma^\mu e^+_L] \\ & + \frac{g}{\sqrt{2}}X^{\alpha'}_\mu[-\epsilon_{\alpha\beta\gamma}\bar{d}^{c\gamma}_L\gamma^\mu d^\beta_L - \bar{u}_{R\alpha}\gamma^\mu\nu^c_R - \bar{u}_{L\alpha}\gamma^\mu\nu^c_L] \\ & + \frac{g}{\sqrt{2}}Y^{\alpha'}_\mu[\epsilon_{\alpha\beta\gamma}\bar{d}^{c\gamma}_L\gamma^\mu u^\beta_L - \bar{u}_{R\alpha}\gamma^\mu e^+_R - \bar{d}_{L\alpha}\gamma^\mu\nu^c_L] \\ & + \frac{g}{\sqrt{2}}X^\alpha_{s\mu}[\bar{d}_{L\alpha}\gamma^\mu e^-_L + \bar{d}_{R\alpha}\gamma^\mu e^-_R + \bar{u}_{L\alpha}\gamma^\mu\nu_L + \bar{u}_{R\alpha}\gamma^\mu\nu_R] \end{aligned} \tag{2.10}$$

where α, β and γ are colour indices. The X_s does not contribute to nucleon decay except by mixing with the X' because there is no vertex qqX_s. We consider baryon violating processes mediated by the gauge fields X, X', Y and Y' of SO(10). In previous papers [36, 51], baryon number violating processes resulting from the coupling to scalar condensates in the string core have been considered. In our SO(10) model we do not have such a coupling.

2.3 Scattering of fermions from the abelian string

2.3.1 The scattering cross-section

Here, we will briefly review the two methods used to calculate the scattering cross-section. The first is a quantum mechanical treatment. From the fermionic Lagrangian L_F, we derive the equations of motion inside and outside the string core. We then find solutions to the equations of motion inside and outside the string core and we match our solutions at the string core. Considering incoming plane waves of pure quarks, we then calculate the scattering amplitude. The matching conditions together with the scattering amplitude enable us to calculate the elastic and inelastic scattering cross-sections. The second method is a quantised one, where one calculates the geometrical cross-section $(\frac{d\sigma}{d\Omega})_{geom}$, i.e. using free fermions spinors ψ_{free}. The catalysis cross-section is enhanced by a factor $\mathcal{A}^4$ over the geometrical cross-section,

$$\sigma_{inel} = \mathcal{A}^4\,(\frac{d\sigma}{d\Omega})_{geom} \tag{2.11}$$

where the amplification factor $\mathcal{A}$ is defined by,

$$\mathcal{A} = \frac{\psi(R)}{\psi_{free}(R)} \,. \tag{2.12}$$

where R is the radius of the string, $R \sim \eta^{-1}$. This method has been applied in Refs. [36] and [59].

2.3.2 The equations of motion

The fermionic part of the Lagrangian L_F is given in terms of 16 dimensional spinors as defined in Appendix A. We shall consider here only one family, the electron family. The generalisation to three families is straight-forward. The fermionic Lagrangian for only one family,

$$L_F = L_F^{(e)} = \bar{\Psi}_{16}\gamma^\mu D_\mu \Psi_{16} + L_M + L_x \tag{2.13}$$

where L_M is the mass term and L_x is the Lagrangian describing quark to lepton transitions through the X, X', Y, Y' and X_s gauge bosons in SO(10) and given by (equation 2.10). The covariant derivative is given by $D_\mu = \partial_\mu - ieA_{\mu,str}\tau_{str}$ where $A_{\mu,str}$ is the gauge field forming the string and τ_{str} is the string generator given by Eq. (2.4). Therefore, since τ_{str} is diagonal, there will be no mixing of fermions around the string. The Lagrangian L_F will split in a sum of eight terms, one for each fermion of the family. In terms of 4-spinors, this is

$$L_F = \sum_{i=1}^{8} L_f^i + L_x \tag{2.14}$$

where $L_f^i = i\bar{\psi}_L^i \gamma^\mu D_\mu^L \psi_L^i + i\bar{\psi}_L^{c,i}\gamma^\mu D_\mu^{Lc}\psi_L^{c,i} + L_m^i$, and i runs over all fermions of the given family. One can show that $i\bar{\psi}_L^{c,i}\gamma^\mu D_\mu^L \psi_L^{c,i} = i\bar{\psi}_R^i\gamma^\mu D_\mu^R \psi_R^i$ and $\tau_{str}^{Lc\,i} = \tau_{str}^{R\,i}$. Finally, L_x is given by Eq. (2.10). It is easy to generalise to more families.

From Eqs. (2.14) and (2.10) we derive the equations of motions for the fermionic fields. We take the fermions to be massless inside and outside the string core. This a relevant assumption since our methods apply for energies above the confinement scale. We consider the case of free quarks scattering from the string and coupling with electrons inside the string core. Outside the string core, the fermions feel the presence of the string only by the presence of the gauge field.We are interested in the elastic cross sections for all fermions and in the cross-section for these quark decaying into electron. The fermionic Lagrangian given by equations (2.14) and (2.10) becomes,

$$\begin{aligned} L_F(e,q) \;\; = \;\; & i\bar{e}_L\gamma^\mu D_\mu^{e,L} e_L + i\bar{e}_R\gamma^\mu D_\mu^{e,R} e_R \\ & + i\bar{q}_L\gamma^\mu D_\mu^{q,L} q_L + i\bar{q}_R\gamma^\mu D_\mu^{q,R} q_R \\ & - \frac{gG^\mu}{2\sqrt{2}}\bar{q}_L\gamma_\mu e_L^+ - \frac{gG'^\mu}{2\sqrt{2}}\bar{q}_R\gamma_\mu e_R^+ + H.C. \end{aligned} \tag{2.15}$$

giving the following equations of motion,

$$\begin{aligned}
i\gamma^\mu D_\mu^{e,L} e_L + \frac{gG'_\mu}{2\sqrt{2}}\gamma^\mu q_L^c &= 0 \\
i\gamma^\mu D_\mu^{e,R} e_R + \frac{gG_\mu}{2\sqrt{2}}\gamma^\mu q_R^c &= 0 \\
i\gamma^\mu D_\mu^{q^c,L} q_L^c + \frac{gG'_\mu}{2\sqrt{2}}\gamma^\mu e_L^- &= 0 \\
i\gamma^\mu D_\mu^{q^c,R} q_R^c + \frac{gG_\mu}{2\sqrt{2}}\gamma^\mu e_R^- &= 0
\end{aligned} \tag{2.16}$$

which are valid everywhere. The covariant derivatives $D_\mu^{e,(L,R)} = \partial_\mu + ieA_{\mu,str}\tau_{str}^{e,(L,R)}$ and $D_\mu^{q^c,(L,R)} = \partial_\mu + ieA_{\mu,str}\tau_{str}^{q,(L,R)}$. We have $\tau_{str}^{R,u} = \tau_{str}^{L,u} = \tau_{str}^{L,e} = \tau_{str}^{L,d} = \frac{1}{10}$ and $\tau_{str}^{R,e} = \tau_{str}^{R,d} = \frac{-3}{10}$ together with $\tau_{str}^{Lc,\,i} = \tau_{str}^{R,\,i}$ and $\tau_{str}^{L,\,i} = \tau_{str}^{Rc,\,i}$. G_μ and G'_μ stand for X_μ, X'_μ, Y'_μ or Y'_μ, depending on the chosen quark.

Since these equations involve quarks and lepton mixing, we do not find independent solution for the quark and lepton fields. However, we can solve these equations taking linear combinations of the the quark and lepton fields, $q_L^c \pm e_L$ and $q_R^c \pm e_R$. In this case, the effective gauge fields are

$$e\left(A_{\mu,str}\tau_{str}^{f_L} \pm G_\mu\right) \tag{2.17}$$

and

$$e\left(A_{\mu,str}\tau_{str}^{f_R} \pm G'_\mu\right) \tag{2.18}$$

respectively.

In order to make the calculations easier, we use a top-hat theta component for G and G' within the string core, since Perkins *et al.*[36] have shown that the physical results are insensitive to the core model used for the gauge fields mediating baryon violating processes.

2.3.3 The external solution

Outside the string core, the gauge field of the string $A_{\mu,str}$ has only, from equations 2.6 and 2.8, a non vanishing component $A_\theta = \frac{1}{er}\tau_{str}$, and the effective gauge fields G and G' are set to zero. Therefore the equations of motion (2.16) for $r > R$ become,

$$\begin{aligned}
i\gamma^\mu D_\mu^{e,L} e_L &= 0 \\
i\gamma^\mu D_\mu^{e,R} e_R &= 0 \\
i\gamma^\mu D_\mu^{u,L} q_L^c &= 0 \\
i\gamma^\mu D_\mu^{u,R} q_R^c &= 0
\end{aligned} \tag{2.19}$$

where the covariant derivatives $D_\mu^{e,(L,R)} = \partial_\mu + ieA_{\mu,str}\tau_{str}^{e,(L,R)}$ and $D_\mu^{q^c,(L,R)} = \partial_\mu + ieA_{\mu,str}\tau_{str}^{q,(L,R)}$.

We take the usual Dirac representation $e_L = (0, \xi_e)$, $e_R = (\chi_e, 0)$, $q_L^c = (0, \xi_{q^c})$ and $q_R^c = (\chi_{q^c}, 0)$ and the mode decomposition for the spinors ξ_{q^c}, ξ_e, χ_{q^c} and χ_e,

$$
\begin{aligned}
\chi_{(e,q^c)}(r,\theta) &= \sum_{n=-\infty}^{n=+\infty} \begin{pmatrix} \chi^n_{1\,(e,q^c)}(r) \\ i \;\; \chi^n_{2\,(e,q^c)}(r)\, e^{i\theta} \end{pmatrix} e^{in\theta} \\
\xi_{(e,q^c)}(r,\theta) &= \sum_{n=-\infty}^{n=+\infty} \begin{pmatrix} \xi^n_{1\,(e,q^c)}(r) \\ i \;\; \xi^n_{2\,(e,q^c)}(r)\, e^{i\theta} \end{pmatrix} e^{in\theta} \, .
\end{aligned} \tag{2.20}
$$

From appendix B.1 we see that the fields $\xi^n_{1,(e,q^c)}$, $\xi^n_{2,(e,q^c)}$, $\chi^n_{1,(e,q^c)}$ and $\chi^n_{2,(e,q^c)}$ satisfy Bessel equations of order $n - \tau_{str}^{R\,(e,q^c)}$, $n + 1 - \tau_{str}^{R\,(e,q^c)}$, $n - \tau_{str}^{L\,(e,q^c)}$ and $n - \tau_{str}^{R\,(e,q^c)}$ respectively. The external solution becomes,

$$
\begin{pmatrix} \xi_{(e,q^c)}(r,\theta) \\ \chi_{(e,q^c)}(r,\theta) \end{pmatrix} = \tag{2.21}
$$

$$
\sum_{n=-\infty}^{n=+\infty} \begin{pmatrix}
 & (v_n^{(e,q^c)} J_{n-\tau_{str}^{R\,(e,q^c)}}(\omega r) & + & v_n^{(e,q^c)\prime} J_{-(n-\tau_{str}^{R\,(e,q^c)})}(\omega r)) & e^{in\theta} \\
i & (v_n^{(e,q^c)} J_{n+1-\tau_{str}^{R\,(e,q^c)}}(\omega r) & - & v_n^{(e,q^c)\prime} J_{-(n+1-\tau_{str}^{R\,(e,q^c)})}(\omega r)) & e^{i(n+1)\theta} \\
 & (w_n^{(e,q^c)} J_{n-\tau_{str}^{L\,(e,q^c)}}(\omega r) & + & w_n^{(e,q^c)\prime} J_{-(n-\tau_{str}^{L\,(e,q^c)})}(\omega r)) & e^{in\theta} \\
i & (w_n^{(e,q^c)} J_{n+1-\tau_{str}^{L\,(e,q^c)}}(\omega r) & - & w_n^{(e,q^c)\prime} J_{-(n+1-\tau_{str}^{L\,(e,q^c)})}(\omega r)) & e^{i(n+1)\theta}
\end{pmatrix} .
$$

Therefore, outside the string core, we have got independent solutions for the quark and electron fields.

2.3.4 The internal solution

Inside the string core, the gauge field of the string, A_μ, is set to zero whereas G_θ and G'_θ take the value $2\sqrt{2}A$ and $2\sqrt{2}A'$ respectively. Therefore, the equations of motion (2.16) become,

$$
\begin{aligned}
i\gamma^\mu \partial_\mu e_L + \frac{gG'_\mu}{2\sqrt{2}} \gamma^\mu q_L^c &= 0 \\
i\gamma^\mu \partial_\mu e_R + \frac{gG_\mu}{2\sqrt{2}} \gamma^\mu q_R^c &= 0 \\
i\gamma^\mu \partial_\mu q_L^c + \frac{gG'_\mu}{2\sqrt{2}} \gamma^\mu e_L^- &= 0 \\
i\gamma^\mu \partial_\mu q_R^c + \frac{gG_\mu}{2\sqrt{2}} \gamma^\mu e_R^- &= 0 \, .
\end{aligned} \tag{2.22}
$$

Since these equations of motions involve quark-lepton mixing, there are no independent solutions for the quarks and electron fields. However, we get solutions for the fields $\rho^\pm$ and $\sigma^\pm$ which are linear combinations of the quarks and electron fields,

$$
\rho^\pm = \chi_{q^c} \pm \chi_e \tag{2.23}
$$

and

$$
\sigma^\pm = \xi_{q^c} \pm \xi_e \, . \tag{2.24}
$$

Using the mode decomposition (2.20) for the fields $\rho^{\pm}$ and $\sigma^{\pm}$, the internal solution becomes,

$$\begin{pmatrix} \rho^{\pm}_{n1}\, e^{in\theta} \\ i \quad \rho^{\pm}_{n2}\, e^{i(n+1)\theta} \\ \sigma^{\pm}_{n1}\, e^{in\theta} \\ i \quad \sigma^{\pm}_{n2}\, e^{i(n+1)\theta} \end{pmatrix} \tag{2.25}$$

where $\rho^{\pm}_{n1}$ and $\rho^{\pm}_{n2}$ and $\sigma^{\pm}_{n1}$ and $\sigma^{\pm}_{n2}$ are the upper and lower components of the fields $\rho^{\pm}$ and $\sigma^{\pm}$ respectively. They are given in terms of hyper-geometric functions. From appendix B.2 we get,

$$\rho^{\pm}_{n1} = (kr)^{|n|} e^{-ikr} \sum_{j=0}^{n=+\infty} \alpha^{\pm}_j \frac{(2ikr)^j}{j!} \tag{2.26}$$

where $k^2 = w^2 - (eA)^2$, $e = \frac{g}{2\sqrt{2}}$. $\alpha^{\pm}_{j+1} = \frac{(a^{\pm}+j)}{(b+p)} \alpha^{\pm}_j$ with $a^{\pm} = \frac{1}{2} + |n| \pm \frac{eA(2n+1)}{2ik}$ and $b = 1 + 2|n|$. $\rho^{\pm}_{n2}$ can be obtained using the coupled equation (B.5.2) of appendix B.2. We find,

$$\rho^{\pm}_{n2} = -\frac{1}{\omega} (kr)^{|n|} e^{-ikr} \sum_{j=0}^{n=+\infty} \alpha^{\pm}_j \frac{(2ikr)^j}{j!} \; (\frac{|n| - n}{r} - ik + \frac{j}{r} \pm eA) \; . \tag{2.27}$$

We get similar hyper-geometric functions for the fields $\sigma^{\pm}_{n1}$ and $\sigma^{\pm}_{n2}$.

2.3.5 Matching at the string core

From now on, we will do calculations for the right-handed fields, the calculations for the left-handed ones being straight-forward. Once we have our internal and external solutions, we match them at the string core. We must take the same linear combinations of the quark and lepton fields outside and inside the core, and must have continuity of the solutions at $r = R$. The continuity of the solutions at $r = R$ implies,

$$(\chi^n_{1,q} \pm \chi^n_{1,e})^{out} = \rho^{\pm\, in}_{n1} \tag{2.28}$$

$$(\chi^n_{2,q} \pm \chi^n_{2,e})^{out} = \rho^{\pm\, in}_{n2} \; . \tag{2.29}$$

Nevertheless, we will have discontinuity of the first derivatives,

$$(\frac{d}{dr} \mp eA)\, \rho^{\pm\, in}_{n2} = (\frac{d}{dr} - \frac{\tau^{R\,(e,q^c)}_{str}}{R})\, (\chi^n_{2,q} \pm \chi^n_{2,e})^{out} \tag{2.30}$$

$$(\frac{d}{dr} \pm eA)\, \rho^{\pm\, in}_{n1} = (\frac{d}{dr} + \frac{\tau^{R\,(e,q^c)}_{str}}{R})\, (\chi^n_{1,q} \pm \chi^n_{1,e})^{out} \; . \tag{2.31}$$

These equations lead to a relation between the coefficients of the Bessel functions for the external solution, as derived in Appendix B.3,

$$\frac{v^{q'}_n \pm v^{e'}_n}{v^q_n \pm v^e_n} = \frac{w\, \lambda^{\pm}_n J_{n+1-\tau_R}(\omega R) + J_{n-\tau_R}(\omega R)}{w\, \lambda^{\pm}_n J_{-(n+1-\tau_R)}(\omega R) + J_{-(n-\tau_R)}(\omega R)} \tag{2.32}$$

where

$$\lambda^{\pm}_n = \frac{\sum_{j=0}^{n=+\infty} \alpha^{\pm}_j \frac{(2ikr)^j}{j!}}{\sum_{j=0}^{n=+\infty} \alpha^{\pm}_j \frac{(2ikr)^j}{j!} (\frac{|n|-n}{r} - ik + \frac{j}{r} \pm eA)} \; . \tag{2.33}$$

The relations (2.32) and (2.33) are the matching conditions at $r = R$.

2.3.6 The scattering amplitude

In order to calculate the scattering amplitude, we match our solutions to an incoming plane wave plus an outgoing scattered wave at infinity. However, since the internal solution, and therefore the matching conditions at $r = R$, are given in terms of linear combinations of quarks and leptons, we consider incoming waves of such linear combinations. Let $f_n^{\pm}$ denote the scattering amplitude for the mode n, f_n^+ if we consider the scattering of (quarks + electrons) and f_n^- if we consider the scattering of (quarks - electrons). Then the matching conditions at infinity are,

$$(-i)^n \begin{pmatrix} J_n \\ iJ_{n+1}\, e^{i\theta} \end{pmatrix} + \frac{f_n^{\pm} e^{ikr}}{\sqrt{r}} \begin{pmatrix} 1 \\ i \quad e^{i\theta} \end{pmatrix} =$$

$$\begin{pmatrix} (v_n^q \pm v_n^e) J_{n-\tau_R} & + & (v_n^{q'} \pm v_n^{e'}) J_{-(n-\tau_R)} & \\ i \quad ((v_n^q \pm v_n^e) J_{n+1-\tau_R} & + & (v_n^{q'} \pm v_n^{e'}) J_{-(n+1-\tau_R)}) & e^{i\theta} \end{pmatrix} . \qquad (2.34)$$

Using then the large r forms for the Bessel functions,

$$J_\mu(\omega r) = \sqrt{\frac{2}{\pi \omega r}} \cos(\omega r - \frac{\mu\pi}{2} - \frac{\pi}{4}) \,, \qquad (2.35)$$

and matching the coefficients of $e^{i\omega r}$ we find,

$$\sqrt{2\pi\omega} f_n^{\pm} e^{i\frac{\pi}{4}} =$$
$$\begin{cases} e^{-in\pi}(e^{i\tau_R \pi} - 1) + (v_n^{q'} \pm v_n^{e'}) e^{i(n-\tau_R)\frac{\pi}{2}} (1 - e^{-2i(n-\tau_R)\pi}) \\ e^{in\pi}(e^{i(n-\tau_R)\pi} - e^{-in\pi}) + (v_n^q \pm v_n^{e\pm}) e^{-i(n-\tau_R)\frac{\pi}{2}} (1 - e^{2i(n-\tau_R)\pi}) \end{cases} . \qquad (2.36)$$

Matching the coefficients $e^{-i\omega r}$, we get relations between the Bessel functions coefficients,

$$(v_n^q \pm v_n^e) = (1 - (v_n^{q'} \pm v_n^{e'}) e^{-i(n-\tau_R)\frac{\pi}{2}})\, e^{-i(n-\tau_R)\frac{\pi}{2}} \,. \qquad (2.37)$$

The relations (2.36), (2.37), (2.32) and (2.33) determine the scattered wave.

2.4 The elastic cross-section

When there is no baryon number violating processes inside the string core, when the gauge fields mediating quark to lepton transitions are set to zero, we have elastic scattering. In this case, the scattering amplitude reduces to,

$$f_n^{elast} = \frac{1}{\sqrt{2\pi\omega}} e^{-i\frac{\pi}{4}} \begin{cases} e^{-in\pi}\, (e^{i\tau_R \pi} - 1) & n \geq 0 \\ e^{in\pi}\, (e^{-i\tau_R \pi} - 1) & n \leq -1 \end{cases} . \qquad (2.38)$$

The elastic cross-section per unit length is given by

$$\sigma_{elast} = | \sum_{n=-\infty}^{+\infty} f_n^{elast} e^{in\theta} |^2 \,. \qquad (2.39)$$

Using the relations $\sum_{n=a}^{+\infty} e^{inx} = \frac{e^{iax}}{1-e^{ix}}$ and $\sum_{n=-\infty}^{b} e^{inx} = \frac{e^{ibx}}{1-e^{-ix}}$, we find the elastic cross-section to be

$$\sigma_{elast} = \frac{1}{2\pi\omega}\frac{\sin^2 \tau_R \pi}{\cos^2 \frac{\theta}{2}} \, . \tag{2.40}$$

This is an Aharonov-Bohm cross-section, and τ_R is the flux in the core of the string.

Now, remember that $\tau_{str}^{Lc,u} = \tau_{str}^{L,u} = \tau_{str}^{Lc,e} = \tau_{str}^{L,d} = \frac{1}{10}$ and $\tau_{str}^{L,e} = \tau_{str}^{Lc,d} = \frac{-3}{10}$ and $\tau_{str}^{Lc,\,i} = \tau_{str}^{R,\,i}$ and $\tau_{str}^{L,\,i} = \tau_{str}^{Rc,\,i}$. Hence,

$$\sigma_{elast}^{e_L} = \sigma_{elast}^{d_R} > \sigma_{elast}^{e_R} = \sigma_{elast}^{u_R} = \sigma_{elast}^{d_L} = \sigma_{elast}^{u_L} \, . \tag{2.41}$$

We therefore have a marked asymmetry between fermions. We have got a marked asymmetry between left and right handed electrons, left and right handed down quarks or, since $\sigma_{elast}^{i_{Lc}} = \sigma_{elast}^{i_R}$ and $\sigma_{elast}^{i_{Rc}} = \sigma_{elast}^{i_L}$, between left handed particle and antiparticle, respectively right handed, for the electron and the down quark. But we have equal cross sections for right handed particles and left handed antiparticles for the electrons and the down quark, and equal cross sections for both left handed and right handed up quarks an anti-quarks. This is a marked feature of grand unified theories. If cosmic strings are found it may be possible to use this asymmetry to identify the underlying gauge symmetry.

2.5 The inelastic cross-section

The gauge fields X, X', Y and Y' are now 'switched on'. In this case we are calculating the baryon number violating cross-section. If we consider identical beams of incoming pure ρ^+ and ρ^-, recalling that $\rho^\pm = \chi_{q^c} \pm \chi_e$, this will ensure that we will have an incoming beam of pure quark. Therefore, the scattering amplitude for the quark field is given by half the difference of f_n^+ and f_n^-, and the scattering amplitude for the electron field is given by half the sum of f_n^+ and f_n^-. From equation (2.36) we get,

$$\frac{1}{2}\sqrt{2\pi\omega}\,(f_n^+ - f_n^-)\, e^{i\frac{\pi}{4}} = v_n^e\, e^{-i(n-\tau_R)\frac{\pi}{2}}\,(1 - e^{-\tau_R 2\pi}) \, . \tag{2.42}$$

The inelastic cross-section for the quark field is given by,

$$\sigma_{inel} = |\sum_{n=-\infty}^{+\infty} (f_n^+ - f_n^-)\, e^{in\theta}|^2. \tag{2.43}$$

Hence, from equation (2.42),

$$\sigma_{inel} \sim \frac{1}{\omega}|\sum_{n=-\infty}^{+\infty} v_n^e\, e^{-in(\frac{\pi}{2}-\theta)}|^2 \, . \tag{2.44}$$

Using equations (2.32), (2.33) and (2.37), we find,

$$v_n^e = \frac{e^{i(n-\tau_R)\frac{\pi}{2}}}{2}\Big(\frac{1}{\delta_n^+ + e^i(n-\tau_R\pi)} - \frac{1}{\delta_n^- + e^i(n-\tau_R\pi)}\Big) \tag{2.45}$$

where

$$\delta_n^{\pm} = \frac{w\,\lambda_n^{\pm} J_{n+1-\tau_R}(\omega R) + J_{n-\tau_R}(\omega R)}{w\,\lambda_n^{\pm} J_{-(n+1-\tau_R)}(\omega R) + J_{-(n-\tau_R)}(\omega R)} \tag{2.46}$$

and $\lambda^{\pm}$ are given by equations (2.33). Equations (2.44), (2.45) and (2.46) determine the inelastic cross-section.This is given in terms of a power series. However, using small argument expansions for Bessel functions, we conclude that this power series involves always one dominant term, the other terms being suppressed by a factor $(\omega R)^n$ where n is an integer such that $n \geq 1$. Therefore the inelastic cross-section involves one dominant mode, the other modes being exponentially suppressed. If d denotes the dominant mode we get $\sigma_{inel} \sim \frac{1}{\omega}\,|v_d^e|^2$. The value of the dominant mode depends on the sign of the the fractional flux τ_{str}. Our results can be summarised as follow.

For $0 < \tau_R < 1$, the mode $n = 0$ is enhanced, and the other modes are exponentially suppressed. Hence,

$$\sigma_{inel} \sim \frac{1}{\omega}\,|v_0^e|^2 \;. \tag{2.47}$$

Using small argument expansions for Bessel functions, this yields

$$\sigma_{inel} \sim \frac{1}{\omega}\,(eAR)^2\,(\omega R)^{4(1-\tau_R)} \tag{2.48}$$

where A is the value of the gauge field inside the string core, e is the gauge coupling constant, and $R \sim \eta$, η being the the grand unified scale$\sim 10^{15} GeV$. The greater amplification occurs for $eAR \sim 1$, giving $\sigma_{inel} \sim \frac{1}{\omega}\,(\omega R)^{4(1-\tau_R)}$.

For $-1 < \tau_R < 0$, the mode $n = -1$ is enhanced, and the other modes are exponentially suppressed. Hence,

$$\sigma_{inel} \sim \frac{1}{\omega}\,|v_{-1}^e|^2 \;. \tag{2.49}$$

Using small argument expansions for Bessel functions, this yields

$$\sigma_{inel} \sim \frac{1}{\omega}\,(eAR)^2\,(\omega R)^{4(1+\tau_R)} \;. \tag{2.50}$$

The greater amplification occurs for $eAR \sim 1$, giving $\sigma_{inel} \sim \frac{1}{\omega}\,(\omega R)^{4(1+\tau_R)}$. Thus, the baryon number violating cross-section is not a strong interaction cross-section but is suppressed by a factor depending on the grand unified scale $\eta \sim R^{-1} \sim 10^{15} GeV$. The baryon number violation cross-sections are very small. For u_L and d_L we obtain,

$$\sigma_{inel} \sim \frac{1}{\omega}\,(\omega R)^{3.6} \;. \tag{2.51}$$

Whereas for d_R we get,

$$\sigma_{inel} \sim \frac{1}{\omega}\,(\omega R)^{2.8} \;. \tag{2.52}$$

Here again we have a marked asymmetry between left and right handed fields. We find an indeterminate solution for the left-conjugate up quark because its phase around the string $(\frac{1}{10})$ differs from the phase of the left-handed electron $(\frac{-3}{10})$ by a fractional value different from a half.

2.6 The second quantised cross-section

We now derive the baryon number violating cross-sections using the perturbative method introduced in Sec. 2.3.1.

Firstly, we calculate the geometrical cross-section. This is the cross-section for free fields ψ_{free}, where ψ_{free} is a 2-spinor. In the case of gauge fields mediating catalysis it is given by,

$$(\frac{d\sigma}{d\Omega})_{geom} = \frac{1}{\omega}\,(\omega R)^4\,(eAR)^2 \tag{2.53}$$

where ω is the energy of the massless field ψ_{free}, A is the value of the gauge field mediating quark to lepton transitions, e is the gauge coupling constant and R is the radius of the string with $R \sim \eta^{-1}$ with $\eta \sim 10^{15}$ GeV.

The second step is to calculate the amplification factor $\mathcal{A} = \frac{\psi}{\psi_{free}}$, ψ and ψ_{free} being two 2-spinors. The catalysis cross-section is enhanced by a factor $\mathcal{A}^4$ over the geometrical cross-section,

$$\sigma_{inel} \sim \mathcal{A}^4\,(\frac{d\sigma}{d\Omega})_{geom}\ . \tag{2.54}$$

We now use the results of sections 2.3.3, 2.3.4 and 2.3.5 where we have solved the equations of motion for the fields ψ and calculated the matching conditions. Using equation (2.21), we get the wave function ψ at the string core, and for the mode n,

$$\psi^n = \left(\begin{array}{cccc} ((v_n^q \pm v_n^e)\,J_{n-\tau_{str}}(\omega R) & + & (v_n^{q'} \pm v_n^{e'})\,J_{-(n-\tau_{str})}(\omega R)) & e^{in\theta} \\ i\ \ ((v_n^q \pm v_n^e)\,J_{n+1-\tau_{str}}(\omega R) & + & (v_n^{q'} \pm v_n^{e'})\,J_{-(n+1-\tau_{str})}(\omega R)) & e^{i(n+1)\theta} \end{array}\right) \tag{2.55}$$

Using equations (2.32) and (2.33) and using small argument expansions for Bessel functions, we conclude that for $n \geq 0$, $(v_n^q \pm v_n^e) \gg (v_n^{q'} \pm v_n^{e'})$, and for $n < 0$, $(v_n^q \pm v_n^e) << (v_n^{q'} \pm v_n^{e'})$. Now, from equation (2.37), we see that one coefficient dominates which will be the $O(1)$. Hence, for $n \geq 0$, $(v_n^q \pm v_n^e) \sim 1$, and for $n < 0$, $(v_n^{q'} \pm v_n^{e'}) \sim 1$. Therefore, using small argument expansions for Bessel functions we get for $n \geq 0$,

$$\psi^n \sim \left(\begin{array}{c} (\omega R)^{n-\tau_{str}} \\ (\omega R)^{n+1-\tau_{str}} \end{array}\right) \tag{2.56}$$

which is to be compared with $\psi_2^{free} \sim 1$ for free spinors. The upper component of the spinor is amplified while the other one is suppressed by a factor $\sim (\omega R)$. For $n < 0$ we have,

$$\psi^n \sim \left(\begin{array}{c} (\omega R)^{-(n-\tau_{str,R})} \\ (\omega R)^{-(n+1-\tau_{str,R})} \end{array}\right) \tag{2.57}$$

Hence we conclude that for $n < 0$ the lower component is amplified while the upper one is suppressed by a factor $\sim \omega R$.

Therefore, for $\tau_{str} = \frac{-3}{10}$, the amplification occurs for the lower component and for the mode $n = -1$. The amplification factor is

$$\mathcal{A} \sim (\omega R)^{\tau_{str}} \tag{2.58}$$

leading to the baryon number violating cross-section,

$$\sigma_{inel} \sim \frac{1}{\omega}\,(eAR)^2\,(\omega R)^{4\,(1+\tau_{str})}\ . \tag{2.59}$$

In the case $\tau_{str} = \frac{1}{10}$, the amplification occurs for the upper component and for the mode $n = 0$. The amplification factor is,

$$\mathcal{A} \sim (\omega R)^{-\tau_{str}} \tag{2.60}$$

leading to the baryon number violating cross-section,

$$\sigma_{inel} \sim \frac{1}{\omega}\,(eAR)^2\,(\omega R)^{4\,(1-\tau_{str})}\ . \tag{2.61}$$

This method shows explicitly which component of the spinor and which mode are enhanced. The results agree with scattering cross-sections derived using the first quantised method.

2.7 Conclusions

We have investigated elastic and inelastic scattering off abelian cosmic strings arising during the phase transition SO(10) $\stackrel{<\phi_{126}>}{\rightarrow}$ SU(5) $\times Z_2$ induced by the vacuum expectation value of a Higgs field in the **126** representation. The cross-sections were calculated using both first quantised and second quantised methods. The results of the two methods are in good agreement.

During the phase transition SO(10) $\rightarrow$ SU(5) $\times Z_2$, only the right-handed neutrino gets a mass. This together with the fact that we are interested in energies above the electroweak scale allows us to consider massless particles.

The elastic cross-sections are found to be Aharonov-Bohm type cross-sections. This is as expected, since we are dealing with fractional fluxes. We found a marked asymmetry between left-handed and right-handed fields for the electron and the down quark fields. But there is no asymmetry for the up quark field. This is a general feature of grand unified theories. If cosmic strings were observed it might be possible to use Aharonov-Bohm scattering to determine the underlying gauge group.

The inelastic cross-sections result from quark to lepton transitions via gauge interactions in the core of the string. The catalysis cross-sections are found to be quite small, and here again we have a marked asymmetry between left and right handed fields. From Eqs. (2.4), (2.59) and (2.61), we see that they are suppressed by a factor $\sim \eta^{-3.6}$ for the left-handed up and down quarks and by a factor $\sim \eta^{-2.8}$ for the right-handed down quarks.

Previous calculations have used a toy model to calculate the catalysis cross-section. Here the string flux could be 'tuned' to give a strong interaction cross-section. In our case the flux is given by the gauge group, and is fixed for each particle species. Hence, we find a strong sensitivity to the grand unified scale. Our small cross-sections make it less likely that grand unified cosmic strings could erase a primordial baryon asymmetry, though they could help generate it [61]. If cosmic strings are observed our scattering results, with the distinctive features for the different particle species, could help tie down the underlying gauge group.

Chapter 3

Constraining supersymmetric SO(10) models through cosmology

3.1 Introduction

Supersymmetric SO(10) models have received much interest in the past ten years. SO(10) is the minimal grand unified gauge group which unifies all kinds of matter, thanks to its 16-dimensional spinorial representation to which all fermions belonging to a single family can be assigned. As mentioned in Chap. 1, the running of the gauge coupling constants measured at LEP in the minimal supersymmetric standard model with supersymmetry broken at 10^3 GeV merge in a single point at 2×10^{16} GeV [19], hence strongly favouring supersymmetric versions of grand unified theories (GUTs). Supersymmetric SO(10) predicts the ratio of the two electroweak Higgs vacuum expectation values, $\tan\beta$, an unknown factor within the minimal supersymmetric standard model, giving $\tan\beta = m_t/m_b$ [62]. Natural doublet-triplet splitting can be achieved in supersymmetric SO(10) via the Dimopoulos-Wilczek mechanism [63]. Note that the doublet-triplet splitting is related to the gauge hierarchy problem: coloured Higgs triplets, which can mediate baryon and lepton number violation, must acquire a vacuum expectation value (VEV) comparable with the GUT scale in order to prevent rapid proton decay, whereas the usual Higgs doublets must acquire a VEV of the order of M_Z. In supersymmetric SO(10), the derivation of fermion masses and mixings can be achieved [64]. The gauge hierarchy problem can be solved [65]. A Z_2 symmetry subgroup of the Z_4 centre of SO(10) can be left unbroken down to low energies, provided only 'safe' representations [44] are used to implement the symmetry breaking from SO(10) down to the standard model gauge group. The Z_2 symmetry can suppress rapid proton decay and provide a cold dark matter candidate, stabilising the lightest superparticle (LSP). Finally, introducing a pair of Higgs fields in the $\mathbf{126} + \overline{\mathbf{126}}$ representations can give a superheavy Majorana mass to the right-handed neutrino, thus providing a hot dark-matter candidate and solving the solar neutrino problem through the Mikheyev-Smirnov-Wolfenstein (MSW) mechanism [40]. Supersymmetric SO(10) is also a good candidate for baryogenesis [23, 66].

Thus, supersymmetric SO(10) is very attractive from a particle physics point of view and can also help to solve some cosmological problems. One would therefore like to be able to select one of the symmetry breaking patterns. Unfortunately, there is considerable freedom in doing so, and the only way out from a particle physics point of view would be from string compactification.

On the other hand, by considering the implications of the symmetry breaking patterns on the standard cosmology and by requiring that the model be consistent with proton lifetime measurements, we can select few of them. As mentioned in Chap. 1, when symmetries spontaneously break down, according to the Kibble mechanism [27], topological defects, such as monopoles, strings or domain walls, may form. Recall that monopoles, because they would be too abundant, and domain walls, because they are too heavy, if present today would dominate the energy density of the universe and lead to a cosmological catastrophe. On the other hand, cosmic strings can explain structure formation and part of the baryon asymmetry of the universe.

We derive below the cosmological constraints on the symmetry breaking schemes of supersymmetric SO(10) down to the standard model due to the formation of topological defects. In Sec. 3.2 we list the possible symmetry breaking pattern involving at most one intermediate symmetry breaking scale. In Sec. 3.3, we review the conditions for the formation of topological defects, giving systematic conditions in supersymmetric SO(10). In Sec. 3.4 we briefly discuss the hybrid inflationary scenario which can naturally arise in supersymmetric SO(10) models, see Sec. 4.2. In sections 3.5, 3.6, 3.7 and 3.8 we give a systematic analysis of the cosmological implications for the different symmetry breaking scenarios listed in Sec. 3.2. We conclude in Sec. 3.9, pointing out the only models not in conflict with the standard cosmology.

3.2 Breaking down to the standard model

In this section, we give a list of all the symmetry breaking patterns from supersymmetric SO(10) down to the standard model, using no more than one intermediate breaking scale. The main differences between supersymmetric and nonsupersymmetric SO(10) models is in the symmetry breaking scales as we shall see and in the choice for the intermediate symmetry groups. In nonsupersymmetric models, at least one intermediate symmetry breaking is needed in order to obtain consistency with the measured value of $\sin^2\theta_w$ and with the gauge coupling constants interpolated to high energy to meet around 10^{15} GeV. On the other hand, in supersymmetric SO(10) models, we can break directly down to the standard model, breaking supersymmetry at $\sim 10^3$ GeV, predicting the measured value of $\sin^2\theta w$ and having the gauge coupling constant joining in a single point at 2×10^{16} GeV.

We shall consider the following symmetry breaking patterns from supersymmetric SO(10) down to the standard model,

$$\mathrm{SO}(10) \overset{M_{\mathrm{GUT}}}{\rightarrow} \mathrm{SU}(5)\times \mathrm{U}(1)_X \overset{M_{\mathrm{G}}}{\rightarrow} SM \tag{3.1}$$
$$\mathrm{SO}(10) \overset{M_{\mathrm{GUT}}}{\rightarrow} \mathrm{SU}(5) \overset{M_{\mathrm{G}}}{\rightarrow} SM \tag{3.2}$$
$$\mathrm{SO}(10) \overset{M_{\mathrm{GUT}}}{\rightarrow} \mathrm{SU}(5)\times \widetilde{\mathrm{U}(1)} \overset{M_{\mathrm{G}}}{\rightarrow} SM \tag{3.3}$$
$$\mathrm{SO}(10) \overset{M_{\mathrm{GUT}}}{\rightarrow} \mathrm{SU}(4)_c\times \mathrm{SU}(2)_L\times \mathrm{SU}(2)_R \overset{M_{\mathrm{G}}}{\rightarrow} SM \tag{3.4}$$
$$\mathrm{SO}(10) \overset{M_{\mathrm{GUT}}}{\rightarrow} \mathrm{SU}(3)_c\times \mathrm{SU}(2)_L\times \mathrm{SU}(2)_R\times \mathrm{U}(1)_{B\text{–}L} \overset{M_{\mathrm{G}}}{\rightarrow} SM \tag{3.5}$$
$$\mathrm{SO}(10) \overset{M_{\mathrm{GUT}}}{\rightarrow} \mathrm{SU}(3)_c\times \mathrm{SU}(2)_L\times \mathrm{U}(1)_R\times \mathrm{U}(1)_{B\text{–}L} \overset{M_{\mathrm{G}}}{\rightarrow} SM \tag{3.6}$$
$$\mathrm{SO}(10) \overset{M_{\mathrm{GUT}}}{\rightarrow} SM \tag{3.7}$$
$$\mathrm{SO}(10) \overset{M_{\mathrm{GUT}}}{\rightarrow} \mathrm{SU}(5)\times \mathrm{U}(1)_X \overset{M_{\mathrm{G}}}{\rightarrow} SM\times Z_2 \tag{3.8}$$
$$\mathrm{SO}(10) \overset{M_{\mathrm{GUT}}}{\rightarrow} \mathrm{SU}(5)\times Z_2 \overset{M_{\mathrm{GUT}}}{\rightarrow} SM\times Z_2 \tag{3.9}$$
$$\mathrm{SO}(10) \overset{M_{\mathrm{GUT}}}{\rightarrow} \mathrm{SU}(5)\times \widetilde{\mathrm{U}(1)} \overset{M_{\mathrm{G}}}{\rightarrow} SM\times Z_2 \tag{3.10}$$
$$\mathrm{SO}(10) \overset{M_{\mathrm{GUT}}}{\rightarrow} \mathrm{SU}(4)_c\times \mathrm{SU}(2)_L\times \mathrm{SU}(2)_R \overset{M_{\mathrm{G}}}{\rightarrow} SM\times Z_2 \tag{3.11}$$
$$\mathrm{SO}(10) \overset{M_{\mathrm{GUT}}}{\rightarrow} \mathrm{SU}(3)_c\times \mathrm{SU}(2)_L\times \mathrm{SU}(2)_R\times \mathrm{U}(1)_{B\text{–}L} \overset{M_{\mathrm{G}}}{\rightarrow} SM\times Z_2 \tag{3.12}$$
$$\mathrm{SO}(10) \overset{M_{\mathrm{GUT}}}{\rightarrow} \mathrm{SU}(3)_c\times \mathrm{SU}(2)_L\times \mathrm{U}(1)_R\times \mathrm{U}(1)_{B\text{–}L} \overset{M_{\mathrm{G}}}{\rightarrow} SM\times Z_2 \tag{3.13}$$
$$\mathrm{SO}(10) \overset{M_{\mathrm{GUT}}}{\rightarrow} SM\times Z_2 \tag{3.14}$$

where SM stands for the standard model gauge group $\mathrm{SU}(3)_c\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$. In models (3.1) to (3.7), supersymmetry must be broken at $\sim 10^3$ GeV, and the symmetry group $\mathrm{SU}(3)_c\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ is broken by the usual Higgs mechanism down to $\mathrm{SU}(3)_c\times\mathrm{U}(1)_Q$ at $\sim M_{\mathrm{Z}}$. The process by which supersymmetry is broken is not considered. In models (3.8) to (3.14), supersymmetry must also be broken at $M_{\mathrm{s}}\sim 10^3$ GeV, and the group symmetry $\mathrm{SU}(3)_c\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y\times Z_2$ is broken by the usual Higgs mechanism down to $\mathrm{SU}(3)_c\times\mathrm{U}(1)_Q\times Z_2$ at $\sim M_{\mathrm{Z}}$. In the latter cases, the Z_2 symmetry remains unbroken down to low energy, and acts as matter parity. It preserves large values for the proton lifetime and stabilises the lightest supersymmetric particle (LSP), thus providing a good hot dark matter candidate.

In order to satisfy LEP data, we must have $M_{\mathrm{GUT}}\sim M_{\mathrm{G}}$ (see Langacker and Luo in Ref. [19]). For nonsupersymmetric models, the value of the B–L symmetry breaking scale is anywhere between 10^{10} and $10^{13.5}$ GeV [67]. For the supersymmetric case it is around $10^{15}-10^{16}$ GeV. Indeed, the scale M_{G} is fixed by the unification of the gauge couplings, and in the absence of particle threshold corrections is $M_{\mathrm{G}}\sim 10^{16}$ GeV [19]. But, as in the nonsupersymmetric case, threshold corrections can induce uncertainties of a factor $10^{\pm 1}$ GeV. These corrections vary with the intermediate subgroup considered, but in any cases, we can assume that $M_{\mathrm{G}}\sim 10^{15}-10^{16}$ GeV. The scale M_{GUT} must be greater than the unified scale M_{G} and below the Planck scale, therefore we must have $10^{19}\,\mathrm{GeV}\geq M_{\mathrm{GUT}}\geq 10^{15}-10^{16}$ GeV.

In order to simplify the notation, we shall use

$$4_c 2_L 2_R \equiv \mathrm{SU}(4)_c\times\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R \tag{3.15}$$
$$3_c 2_L 2_R 1_{B-L} \equiv \mathrm{SU}(3)_c\times\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R\times\mathrm{U}(1)_{B-L} \tag{3.16}$$

$$3_c 2_L 1_R 1_{B-L} \equiv \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_R \times \mathrm{U}(1)_{B-L} \tag{3.17}$$

$$3_c 2_L 1_Y (Z_2) \equiv \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y (\times Z_2) \tag{3.18}$$

$$3_c 1_Q (Z_2) \equiv \mathrm{SU}(3)_c \times \mathrm{U}(1)_Q (\times Z_2) \tag{3.19}$$

3.3 Topological defect formation in supersymmetric models

In this section, we show that the conditions for topological defect formation well known in non supersymmetric theories (see Ref. [27] and also Chap. 1) are not affected by the presence of supersymmetry. We then study the formation of hybrid defects, such as monopoles connected by strings or domain walls bounded by strings, which can arise in SO(10) models, particularly looking at their cosmological impact [68, 69].

3.3.1 Defect formation in supersymmetric models

We study here the conditions for defect formation in supersymmetric models. We show that the conditions for topological defect formation in nonsupersymmetric theories [27], are not affected by the presence of supersymmetry.

In nonsupersymmetric theories, the conditions for topological defect formation during the spontaneous symmetry breaking of a nonsupersymmetric Lie group G to a nonsupersymmetric Lie group H are well known; they are associated with the connection of the vacuum manifold $\frac{\mathrm{G}}{\mathrm{H}}$ [27]. Now one may worry about the non Lie nature of the superalgebra. Fortunately, it has been shown [70] that the superalgebra is Lie admissible and that the infinitesimal transformations of the superalgebra can be exponentiated to obtain a Lie superalgebra. The Lie admissible algebra is an algebraic covering of the Lie algebra, and it was first identified by Albert [71]. It is such a covering that allows a Lie admissible infinitesimal behaviour while preserving the global structure of the Lie group. The graded Lie algebra is Lie admissible and therefore much of the Lie algebra theory may be extended to it with the appropriate modification. In particular, a connected (super)Lie group structure persists [72]. Hence, the formation of topological defects in supersymmetric models will be the same as in nonsupersymmetric ones. Whether or not supersymmetry is broken at the phase transition will not affect the conditions under which topological defects form. These conditions are reviewed in Chap. 1. In this chapter and in the following one, when we denote a group G, we really mean the supersymmetric version of this group, and when we write SO(10) we mean its universal covering group Spin(10) (supersymmetric) which is simply connected.

3.3.2 Hybrid defects

When we have an intermediate breaking scale, we can also get mixed defects. There are two kinds of mixed defects that we can get in supersymmetric SO(10) models; they are monopoles connected by strings and domain walls bounded by strings. Their cosmological evolutions have been studied in a nonsupersymmetric general case [68, 69].

Monopoles connected by strings

In supersymmetric SO(10) models, we can have monopoles connected by strings [69]. If the first phase transition leaves an unbroken U(1) symmetry which later breaks to unity, that is if the breaking pattern proceeds as

$$\mathrm{G} \rightarrow \mathrm{H} \times \mathrm{U}(1)_x \rightarrow \mathrm{H} \tag{3.20}$$

where G and H are both simply connected, then monopoles form at the first phase transition, and then get connected by strings at the following one. Indeed, the second homotopy group $\pi_2(\frac{\mathrm{G}}{\mathrm{H}\times\mathrm{U}(1)}) = \pi_1(\mathrm{H}\times\mathrm{U}(1)) = Z$ indicates the formation of monopoles during the first phase transition in (3.20). These monopoles carry a $\mathrm{U}(1)_x$ magnetic charge, and are topologically unstable. Now the first homotopy group $\pi_1(\frac{\mathrm{H}\times\mathrm{U}(1)}{\mathrm{H}})$ is also non trivial, hence cosmic strings form at the second stage of symmetry breaking in (3.20). The strings connect monopole/antimonopole pairs of the first phase transition [69]. Because the whole system of strings rapidly decays [69], monopoles connected by strings do not seem to affect the standard cosmology in any essential way. On the other hand, if the universe undergoes a period of inflation between the two phase transitions, or if the phase transition leading to the formation of monopoles is itself inflationary, then the picture is very different. The decay of the system of strings is negligible. If the monopoles are inflated beyond the horizon, the strings form according to the Kibble mechanism and their evolution is that of topologically stable cosmic strings [69]. In this class of scenarios, with inflation and cosmic strings, temperature fluctuations in the CBR measured by COBE give constraints on the scale of the phase transition leading to the string formation and on the scalar coupling constant (see next chapter).

Walls bounded by strings

The other kind of topological mixed defect that we can get in SO(10) models is domain walls connected by strings. A first phase transition leaves an unbroken discrete symmetry, and cosmic strings form. At a subsequent phase transition, this discrete symmetry breaks, leading to the formations of domain walls. They are bounded by the strings previously formed. Specifically, consider a symmetry breaking pattern of the form

$$\mathrm{G} \rightarrow \mathrm{H} \times Z_2 \rightarrow \mathrm{H} \tag{3.21}$$

where G and H are both simply connected. The first homotopy group $\pi_1(\frac{\mathrm{G}}{\mathrm{H}\times Z_2}) = \pi_0(\mathrm{H}\times Z_2) = Z_2$; thus, Z_2-strings form during the first phase transition in (3.21), and they are topologically unstable. The discrete Z_2 symmetry breaking leads to the formation of domain walls at the second stage of symmetry breaking bounded by strings of the first phase transition. Such extended objects have been first studied by Kibble et al. [68]. They have shown that, in the nonsupersymmetric case, the cosmological relevance of these mixed objects depends on whether inflation occurs between the time when strings form and the time when the symmetry breaking leading to the formation of these walls occurs. The presence of supersymmetry does not affect the above conclusions. Following Ref. [68], we get the following results. If the transition leading to the formation of the walls takes place without supercooling, the walls lose their energy by friction and disappear in a time $t_d \sim (t_W t_*)^{\frac{1}{2}}$ where t_W is the cosmic time corresponding to the the scale T_W at which the walls form and $t_* = \frac{3\alpha_G \eta_0}{32\pi\eta_3}\frac{M_p^2}{M_{\rm G}^3}$, where η_3 is the effective massless degrees of freedom reflected by the walls and η_0 is the effective number of degrees of freedom in the supersymmetric $3_c 2_L 1_Y (Z_2)$ phase. With $\eta_3 = 33.75$ and $\eta_0 = 228.75$ we find $t_d \sim 10^{-33} - 10^{-36}$ sec for $T_W \sim 10^{15} - 10^{16}$ GeV and the corresponding scale $T_* \sim 10^9 - 10^{12}$ GeV. Therefore these extended objects do not seem to affect the standard cosmology in any essential way. But if there is a period of inflation between the two phase transitions, the strings can be pushed to arbitrarily large scales; the walls form according to the Kibble mechanism and their evolution is that of topologically stable walls. The only difference from topologically stable Z_2-walls is that the walls can now decay by the quantum nucleation of holes bounded by strings. Hole nucleation however is a tunnelling process and is typically suppressed by a large exponential factor. The corresponding decay time is much larger than the time at which the walls come to dominate the universe, thereby upsetting standard cosmology.

3.4 Inflation in supersymmetric SO(10) models

Since SO(10) is simply connected and the standard model gauge group involves an unbroken U(1) symmetry which remains unbroken down to low energy, all symmetry breaking patterns from supersymmetric SO(10) down to the standard model automatically involve the formation of topologically stable monopoles. Even if some monopoles are connected by strings, a large fraction of them will remain stable down to low energy. Hence some mechanism has to be invoked in order to obtain consistency with standard cosmology, such as an inflationary scenario. In this section, we briefly discuss a false vacuum hybrid inflationary scenario which is the most natural mechanism for inflation in global supersymmetric SO(10) models, as will be shown in the next chapter.

The superpotential in the inflaton sector is similar to that studied in Refs. [29, 30]. It involves a scalar field $\mathcal{S}$ singlet under SO(10) and a pair of Higgs fields $\Phi + \overline{\Phi}$ in the $\mathbf{16} + \overline{\mathbf{16}}$ or in the $\mathbf{126} + \overline{\mathbf{126}}$ dimensional representations of SO(10). These Higgs fields are used to lower the rank of SO(10) by one unit, they must get a VEV the order of the GUT scale. If the Z_2 parity is to be kept unbroken, as in models (3.8) to (3.14), a pair of $\mathbf{126} + \overline{\mathbf{126}}$ must be used. The superpotential can be written as

$$W = \alpha \mathcal{S} \overline{\Phi} \Phi - \mu^2 \mathcal{S} \tag{3.22}$$

where μ and α are two positive constants such that $\frac{\mu}{\sqrt{\alpha}} = M_{\rm GUT}$. If the rank of the group is lowered at $M_{\rm G}$, we have $\frac{\mu}{\sqrt{\alpha}} = M_{\rm G}$.

The evolution of the fields is as follows (a complete discussion of the potential in a general supersymmetric case is studied in Ref. [29] and in a specific supersymmetric SO(10) model is studied in the next chapter). The fields take random initial values, just subject to the constraint that the energy density be at the Planck scale. The inflaton field is distinguished from the other fields from the fact that the gradient of the GUT potential with respect to the inflaton field is very small. Therefore the non inflaton fields, except the Φ and $\overline{\Phi}$ fields, will roll very quickly down to their minimum at an approximately fixed value for the inflaton. Inflation occurs as the inflaton rolls slowly down the potential. The symmetry breaking implemented with the $\Phi + \overline{\Phi}$ fields occurs at the end of inflation and associated topological defects are not inflated away, see [29] and Chap. 4.

3.5 SU(5) as intermediate scale

We shall describe in this section the symmetry breaking patterns from supersymmetric SO(10) involving an SU(5) intermediate symmetry. When the intermediate scale involves SU(5) as a subgroup, the scale $M_{\rm G}$ has to be $\sim 10^{16}$ GeV, and consequently the scale $M_{\rm GUT}$ is pushed close to the string compactification scale. SO(10) can break via SU(5) in four different ways. It can break via $\mathrm{SU}(5) \times \mathrm{U}(1)_X$, SU(5) , via $\mathrm{SU}(5) \times \widetilde{\mathrm{U}(1)}$ and via $\mathrm{SU}(5) \times Z_2$.

3.5.1 Breaking via $\mathrm{SU}(5) \times \mathrm{U}(1)_X$

We consider here two symmetry breaking patterns,

$$\begin{aligned} \mathrm{SO}(10) \quad & \stackrel{M_{\rm GUT}}{\to} \mathrm{SU}(5) \times \mathrm{U}(1)_X & (3.23)\\ & \stackrel{M_{\rm G}}{\to} \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y (\times Z_2) & (3.24)\\ & \stackrel{M_{\rm Z}}{\to} \mathrm{SU}(3)_c \times \mathrm{U}(1)_Q (\times Z_2) & (3.25) \end{aligned}$$

with and without the Z_2 symmetry unbroken down to low energy. The latter is necessary to preserve large values for the proton lifetime and to stabilise the LSP. It can arise only if a pair of $(\mathbf{126} + \overline{\mathbf{126}})$ dimensional Higgs representations is used to lower the rank of the group.

The $\mathrm{U}(1)_X$ symmetry commutes with SU(5). The X and Y directions are orthogonal to each other, and thus the $\mathrm{U}(1)_X$ symmetry breaks down to unity at $M_{\rm G}$ (or to Z_2 if a pair of $\mathbf{126} + \overline{\mathbf{126}}$ Higgs fields are used to break $\mathrm{SU}(5) \times \mathrm{U}(1)_X$). This feature is going to affect the formation of topological defects.

The first homotopy group $\pi_1(\mathrm{SU}(5) \times \mathrm{U}(1)_X) = Z$ is non trivial and thus topological monopoles form when SO(10) breaks. They have a mass $M_m \geq 5 \times 10^{17}$ GeV. At the following phase transition the $\mathrm{U}(1)_X$ symmetry breaks to unity (to Z_2) and hence cosmic strings (Z_2-strings) form. They connect monopole-antimonopole pairs previously formed (see section 3.3.2). They have a mass per unit length $\sim 10^{32}$ GeV^2.

When $\mathrm{SU}(5) \times \mathrm{U}(1)_X$ breaks down to $3_c 2_L 1_Y (Z_2)$ new lighter monopoles form. Indeed, since $\mathrm{U}(1)_X$ breaks down to unity (to Z_2) we consider the second homotopy group $\pi_2(\frac{\mathrm{SU}(5)}{3_c 2_L 1_Y})$ to look for monopoles formations at M_{G}. Hence topologically stable monopoles form. They have a mass $M_m \sim 10^{17}$ GeV. They are topologically stable. Their topological charge may change from Y to Q.

Since monopoles form at both phase transitions and since the lighter ones are topologically stable, the inflationary scenario, as in section 3.4, is unable to solve the monopole problem. Hence these two models are inconsistent with observations.

3.5.2 Breaking via SU(5)

Here, SO(10) breaks down to the standard model with intermediate SU(5) symmetry alone. In this case, there is no interest in going to a larger grand unified group. The breaking scheme is

$$\begin{aligned} \mathrm{SO}(10) &\stackrel{M_{\mathrm{GUT}}}{\rightarrow} \mathrm{SU}(5) && (3.26)\\ &\stackrel{M_{\mathrm{G}}}{\rightarrow} \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y && (3.27)\\ &\stackrel{M_{\mathrm{Z}}}{\rightarrow} \mathrm{SU}(3)_c \times \mathrm{U}(1)_Q && (3.28) \end{aligned}$$

which is that of model (3.1). Since SO(10) and SU(5) are both simply connected, no topological defects form during the first stage of symmetry breaking.

The second homotopy group $\pi_2(\frac{\mathrm{SU}(5)}{3_c 2_L 1_Y}) = Z$ hence topological monopoles form when SU(5) breaks down to the standard model. The monopoles carry Y topological charge. The second homotopy group $\pi_2(\frac{\mathrm{SU}(5)}{3_c 1_Q}) = Z$ which shows that the monopoles are topologically stable. They have a mass $M_m \sim 10^{17}$ GeV. Their topological charge may change from Y to Q.

Since the rank of SO(10) is 5 and the rank of SU(5) is 4, if we use an inflationary scenario as described in Sec. 3.4 to solve the monopole problem, the inflaton field will couple to a pair of $\mathbf{16} + \overline{\mathbf{16}}$ Higgs fields representations which will be used used to break SO(10). The monopoles described above will form at the end of inflation, and their density will be high enough to dominate the universe. Hence this model is in conflict with the standard cosmology. It is also inconsistent with the actual data on the proton lifetime.

3.5.3 Breaking via $\mathrm{SU}(5) \times \widetilde{\mathrm{U}(1)}$

More interesting is the breaking via flipped SU(5)

$$\begin{aligned} \mathrm{SO}(10) &\stackrel{M_{\mathrm{GUT}}}{\rightarrow} \mathrm{SU}(5) \times \widetilde{\mathrm{U}(1)} && (3.29)\\ &\stackrel{M_{\mathrm{G}}}{\rightarrow} \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y && (3.30)\\ &\stackrel{M_{\mathrm{Z}}}{\rightarrow} \mathrm{SU}(3)_c \times \mathrm{U}(1)_Q && (3.31) \end{aligned}$$

Note that with flipped SU(5), rather than using SO(10) for the grand unified gauge group, the monopole problem is avoided [73]. The $\widetilde{\mathrm{U}(1)}$ contains part of the electromagnetic gauge group $\mathrm{U}(1)_Q$. The above symmetry breaking can only be implemented in supergravity SO(10) models [73].

The first homotopy group $\pi_1(\mathrm{SU}(5)\times\widetilde{\mathrm{U}(1)}) = Z$ and therefore the first phase transition leads to the formation of topological monopoles when SO(10) breaks. Furthermore, since $\pi_1(3_c2_L1_Y) = \pi_1(3_c1_Q) = Z$ and $\widetilde{\mathrm{U}(1)}$ contains part of the $\mathrm{U}(1)_Y$ and $\mathrm{U}(1)_Q$ symmetries, these monopoles are topologically stable. They have a mass $M_m \geq 5\times 10^{17}$ GeV. They carry $B-L$, and their topological charge may change to Y and then to Q. Embedded cosmic strings form after the second stage of symmetry breaking [79].

We should be able to cure the monopole problem with an hybrid inflationary scenario for supergravity models. Indeed, since the rank of $\mathrm{SU}(5)\times\widetilde{\mathrm{U}(1)}$ is 5, the inflaton field can couple to the Higgs needed to break $\mathrm{SU}(5)\times\widetilde{\mathrm{U}(1)}$, and embedded strings will form at the end of inflation. Hence from a defect point of view the model is interesting, but appears to be inconsistent with the actual data for proton lifetime [44] and does not provide any Majorana mass for the right-handed neutrino. The latter problems are solved if we break $\mathrm{SU}(5)\times\widetilde{\mathrm{U}(1)}$ down to $3_c2_L1_YZ_2$. In that case, a $(\mathbf{126}+\overline{\mathbf{126}})$ dimensional Higgs representation is used to break $\mathrm{SU}(5)\times\widetilde{\mathrm{U}(1)}$. Since the first homotopy groups $\pi_1(\frac{\mathrm{SU}(5)\times\widetilde{\mathrm{U}(1)}}{3_c2_L1_YZ_2}) = Z_2$ and $\pi_1(\frac{\mathrm{SU}(5)\times\mathrm{U}(1)}{3_c1_QZ_2}) = Z_2$, topologically stable Z_2-strings also form. They have a mass per unit length $\sim 10^{32}$ GeV^2.

3.5.4 Breaking via $\mathrm{SU}(5)\times Z_2$

We consider here the breaking of SO(10) via SU(5) with added parity. The symmetry breaking is

$$\mathrm{SO}(10) \overset{M_{\mathrm{GUT}}}{\rightarrow} \mathrm{SU}(5)\times Z_2 \tag{3.32}$$
$$\overset{M_{\mathrm{G}}}{\rightarrow} \mathrm{SU}(3)_c\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y\times Z_2 \tag{3.33}$$
$$\overset{M_{\mathrm{Z}}}{\rightarrow} \mathrm{SU}(3)_c\times\mathrm{U}(1)_Q\times Z_2 \tag{3.34}$$

where the unbroken Z_2 symmetry is a subgroup of the Z_4 centre of SO(10). It plays the role of matter parity. It preserves large values for the proton lifetime and stabilises the LSP; thus, the model is consistent with the actual data on proton decay and provides a good hot dark matter candidate.

Now the fundamental homotopy group $\pi_0(\mathrm{SU}(5)\times Z_2) = Z_2$ and therefore Z_2 cosmic strings form during the first phase transition. They have a mass per unit length $10^{38}\mathrm{GeV}^2 \geq \mu \geq 10^{32}\ \mathrm{GeV}^2$. Since the Z_2 symmetry is kept unbroken down to low energy, these strings remain topologically stable. They have been widely studied in the nonsupersymmetric case, see Ref. [55, 56] and also Chap. 2.

As in section 3.5.2, it is clear that topologically stable monopoles form during the second phase transition with mass $M_m \sim 10^{17}$ GeV. Hence as in section 3.5.2, the model is in contradiction with observations.

We conclude that the only symmetry breaking pattern from SO(10) down to the standard model with intermediate SU(5) symmetry consistent with observations, is

$$\mathrm{SO}(10)\rightarrow\mathrm{SU}(5)\times\widetilde{\mathrm{U}(1)}\rightarrow 3_c2_L1_YZ_2\rightarrow 3_c1_QZ_2 \tag{3.35}$$

where the Z_2 symmetry must be kept unbroken in order to preserve large values for the proton lifetime. The above symmetry breaking can only be implemented in supergravity models.

3.6 Patterns with a left-right intermediate scale

In this section we study the symmetry breaking patterns from supersymmetric SO(10) down to the standard model involving an $SU(2)_L \times SU(2)_R$ intermediate symmetry. These are the symmetry breaking patterns with intermediate $4_c 2_L 2_R (Z_2)$ or $3_c 2_L 2_R 1_{B-L} (Z_2)$ symmetry groups. We show that these models, due the unbroken $SU(2)_L \times SU(2)_R$ symmetry share a property, which can make them cosmologically irrelevant, depending on the Higgs field chosen to implement the symmetry breaking. We then give a full study of the formation of the topological defects in each model.

3.6.1 Domain walls in left-right models

We study here a property shared by the symmetry breaking schemes from SO(10) down to the standard model, with or without unbroken parity Z_2,

$$\mathrm{SO}(10) \overset{M_{\mathrm{GUT}}}{\to} \mathrm{G} \overset{M_G}{\to} 3_c 2_L 1_Y (Z_2) \tag{3.36}$$

where G is either $4_c 2_L 2_R$ or $3_c 2_L 2_R 1_{B-L}$. In these models, the intermediate scale involves an unbroken $SU(2)_L \times SU(2)_R$ symmetry, and consequently the intermediate symmetry group can be invariant under the charge conjugation operator, depending on the Higgs multiplet chosen to break SO(10). The latter leaves an unbroken discrete Z_2^c symmetry which breaks at the following phase transition. In this case, the general symmetry breaking scheme given in Eq. (3.36) should really be written as

$$\mathrm{SO}(10) \overset{M_{\mathrm{GUT}}}{\to} \mathrm{G} \times Z_2^c \overset{M_{\mathrm{G}}}{\to} SM(\times Z_2)\,. \tag{3.37}$$

If $\mathrm{G} = 4_c 2_L 2_R$, the discrete Z_2^c symmetry appears if the Higgs used to break SO(10) is a single 54-dimensional representation [74]. If $\mathrm{G} = 3_c 2_L 2_R 1_{B-L}$ the Z_2^c symmetry appears if a single 210 dimensional Higgs representation is used, with appropriate parameter range in the Higgs potential [75]. The appearance of the discrete Z_2^c symmetry leads to a cosmological problem [68]. Indeed, since Spin(10) is simply connected, $\pi_1(\frac{\mathrm{SO}(10)}{\mathrm{G}\times Z_2^c}) = \pi_0(\mathrm{G} \times Z_2^c) = Z_2$ and therefore Z_2 strings form during the first phase transition associated with the breaking of SO(10). They have a mass per unit length $\sim 10^{32} - 10^{34}\ \mathrm{GeV}^2$. When the discrete Z_2^c symmetry breaks, domain walls form bounded by the strings of the previous phase transition. Some closed walls can also form. As shown in Sec. 3.3.2, these domain walls do not affect the standard cosmology in any essential way. On the other hand, if a period of inflation occurs between the two phase transition, or if the phase transition leading to the walls formation is itself inflationary, then the evolution of the walls is that of topologically stable Z_2 walls. They dominate the universe, destroying the standard cosmology.

3.6.2 Breaking via $4_c 2_L 2_R$

We now consider the symmetry breaking of SO(10) via the Pati-Salam gauge group $4_c 2_L 2_R$ subgroup of SO(10) which later breaks down to the standard model gauge group with or without matter parity

$$\mathrm{SO}(10) \stackrel{M_{\mathrm{GUT}}}{\rightarrow} \mathrm{SU}(4)_c \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \tag{3.38}$$

$$\stackrel{M_{\mathrm{G}}}{\rightarrow} \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y (\times Z_2) \tag{3.39}$$

$$\stackrel{M_{\mathrm{Z}}}{\rightarrow} \mathrm{SU}(3)_c \times \mathrm{U}(1)_Q (\times Z_2) \tag{3.40}$$

with supersymmetry broken at $\simeq 10^3$ GeV and the scales M_{GUT} and M_{G}, respectively, satisfy $M_{pl} \geq M_{\mathrm{GUT}} \geq 10^{16}$ GeV and $M_{\mathrm{G}} \sim 10^{15} - 10^{16}$ GeV. The discrete Z_2 symmetry is kept unbroken if we use a pair of $(\mathbf{126} + \overline{\mathbf{126}})$ dimensional Higgs representation to break $4_c 2_l 2_R$, and is broken if we use a pair of $(\mathbf{16} + \overline{\mathbf{16}})$ dimensional Higgs representation. The unbroken Z_2 symmetry plays the role of matter parity, preserving large values for the proton lifetime and stabilising the LSP. Hence only the model with unbroken Z_2 at low energy is consistent with the actual value for proton lifetime.

If a single 54 dimensional Higgs representation is used to break SO(10), equation (3.40) should really be written as [68]

$$Spin(10) \stackrel{M_{\mathrm{GUT}}}{\rightarrow} \left(\frac{(\mathrm{Spin}(6) \times \mathrm{Spin}(4))}{Z_2}\right) \times Z_2^c \tag{3.41}$$

$$\stackrel{M_{\mathrm{G}}}{\rightarrow} \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y (\times Z_2) \tag{3.42}$$

$$\stackrel{M_{\mathrm{Z}}}{\rightarrow} \mathrm{SU}(3)_c \times \mathrm{U}(1)_Q (\times Z_2) \tag{3.43}$$

where we have explicitly shown the hidden symmetry. A Z_2 symmetry has to be factored out in Eq. (3.41) since Spin(6) and Spin(4) have a non trivial intersection. The overall Z_2^c is generated by the charge conjugation operator; it is unrelated to the previous Z_2 one. Subsequently, the Z_2^c discrete symmetry is broken. If a pair of Higgs fields in the $\mathbf{126} + \overline{\mathbf{126}}$ representation are used to break $4_c 2_l 2_R$, then a new Z_2 symmetry emerges, as described above; it is unrelated to the previous ones. The standard model gauge group is broken with Higgs fields in the **10** dimensional representation of SO(10).

If a single 210-Higgs multiplet is used to break $4_c 2_l 2_R$, with appropriate range in the parameters of the Higgs potential, the Z_2^c does not appear [75].

Monopoles

The non trivial intersection of Spin(6) and Spin(4) leads to the production of superheavy monopoles [69] when SO(10) breaks to $4_c 2_L 2_R$. These monopoles are superheavy with a mass $M_m \geq 10^{17}$ GeV. They are topologically unstable.

Since the second homotopy group $\pi_2(\frac{4_c 2_L 2_R}{3_c 2_L 1_Y (Z_2)}) = Z$ is non trivial, new monopoles form when $4_c 2_L 2_R$ breaks down to the standard model gauge group. They are unrelated to the previous monopoles. Furthermore, since the second homotopy group $\pi_2(\frac{4_c 2_L 2_R}{3_c 1_Q (Z_2)}) = Z$ is also non trivial, these lighter monopoles are topologically stable. They have a mass $M_m \sim 10^{16} - 10^{17}$ GeV. These monopoles form according to the Kibble mechanism, and their density is such that, if present today, they would dominate the energy density of the universe.

Domain walls

If a 54 dimensional Higgs representation is used to break SO(10) down to $4_C 2_L 2_R$, the symmetry breaking is given by Eq. (3.41) which is of the form of Eq. (3.37) with G $= 4_c 2_L 2_R$, so that a discrete Z_2^c symmetry emerges at the intermediate scale. Thus, as shown in Sec. 3.6.1, Z_2-strings form during the first phase transition. (They are unrelated to any of the monopole just discussed above.) During the second stage of symmetry breaking, this Z_2^c breaks, leading to the formation of domain walls which connect the strings previously formed. These walls bounded by strings do not affect the standard cosmology in any essential way. But if there is a period of inflation before the phase transition leading to the walls formation takes place (see section 3.3.2), the walls would dominate the energy density of the universe, leading to a cosmological catastrophe.

Cosmic strings

Now we consider the models where $4_c 2_L 2_R$ breaks down to the standard model gauge group with added Z_2 parity, as in model **8**. Then a new Z_2 symmetry emerges at M_{G}, which is unrelated to the previous ones. Since $\pi_1(\frac{4_c 2_L 2_R}{3_c 2_L 1_Y Z_2}) = Z_2$, Z_2-strings form when $4_c 2_L 2_R$ breaks. They have a mass per unit length $\mu \sim 10^{30} - 10^{32}\,\mathrm{GeV}^2$. Since the Z_2 symmetry is then kept unbroken down to low energy, we break the standard model gauge group with a Higgs 10-plets. The strings are topologically stable down to low energy.

Density perturbations in the early universe and temperature fluctuations in the CBR generated by these strings could be computed.

Solving the monopole problem

In order to solve the monopole problem, we use a hybrid inflationary scenario, as discussed in section 3.4. The rank of both $4_c 2_L 2_R$ and $4_c 2_L 2_R Z_2$ is four. Therefore the inflaton field will couple to a pair of Higgs fields which will break $4_c 2_L 2_R$. The primordial monopoles formed when SO(10) breaks are diluted by the inflation. But then lighter monopoles form at the end of inflation when $4_c 2_L 2_R$ breaks, which are topologically stable. In the case of unbroken Z_2 parity, cosmic strings also form. Monopole creation at this later stage makes the model inconsistent with observations.

If SO(10) is broken with a 54-dimensional Higgs representation, domain walls will form through the Kibble mechanism at the end of inflation, which will dominate the universe, as shown in Sec. 3.6.1, hence leading to a cosmological catastrophe.

We conclude that the model is cosmologically inconsistent with observations. It is inconsistent whether or not the discrete Z_2^c symmetry is unbroken at the intermediate scale.

3.6.3 Breaking via $3_c2_L2_R1_{B-L}$

We can break via $3_c2_L2_R1_{B-L}$ and then down to the standard model with or without the discrete Z_2 symmetry preserved at low energy

$$\begin{aligned} \mathrm{SO}(10) &\overset{M_{\mathrm{GUT}}}{\to} \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B\text{–}L} && (3.44)\\ &\overset{M_{\mathrm{G}}}{\to} \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y(\times Z_2) && (3.45)\\ &\overset{M_{\mathrm{Z}}}{\to} \mathrm{SU}(3)_c \times \mathrm{U}(1)_Q(\times Z_2) && (3.46)\end{aligned}$$

The Z_2 symmetry, which can be kept unbroken down to low energy if only safe representations are used to implement the symmetry breaking, plays the role of matter parity. It preserves large values for the proton lifetime. Hence only models with unbroken Z_2 parity at low energy are consistent with the actual values of proton decay. If SO(10) is broken with a single 210-Higgs multiplet, with the appropriate range of the parameters in the Higgs potential [75], then there appears a discrete Z_2^c symmetry at the intermediate scale which is generated by the charge conjugation operator, and the symmetry breaking really is

$$\begin{aligned} \mathrm{SO}(10) &\overset{M_{\mathrm{GUT}}}{\to} \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B\text{–}L} \times Z_2^c && (3.47)\\ &\overset{M_{\mathrm{G}}}{\to} \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y(\times Z_2) && (3.48)\\ &\overset{M_{\mathrm{Z}}}{\to} \mathrm{SU}(3)_c \times \mathrm{U}(1)_Q(\times Z_2)\,. && (3.49)\end{aligned}$$

The Z_2^c is unrelated to the Z_2 symmetry which can be added to the standard model gauge group in Eqs. (3.48) and (3.49).

If one uses a combination of a 45 dimensional Higgs representation with a 54 dimensional one to break SO(10), then the symmetry breaking is that of equation (3.44), and no discrete symmetry appears as in (3.47) [76]. The rest of the symmetry breaking is implemented with a pair of $\mathbf{16} + \overline{\mathbf{16}}$ Higgs multiplets or with a pair of $\mathbf{126} + \overline{\mathbf{126}}$ Higgs multiplets if matter parity is preserved at low energy. $3_c2_L1_Y$ is broken with a 10-Higgs multiplet.

Monopoles

The first homotopy groups $\pi_1(3_c2_L2_R1_{B-L}) = Z$, $\pi_1(3_c2_L1_Y) = Z$ and $\pi_1(3_c1_Q) = Z$, showing that topologically stable monopoles are produced during the first phase transition from SO(10) down to $3_c2_L2_R1_{B-L}$. They have a mass $M_m \geq 10^{17}$ GeV. These monopoles are in conflict with cosmological observations.

Domain walls

If SO(10) is broken with a single 210-dimensional Higgs representation, then the symmetry breaking is that of Eq. (3.51). Hence, as in the breaking pattern (3.41), the appearance of the discrete Z_2^c symmetry leads to the formation of non stable cosmic strings during the first symmetry breaking and to the formation of domain walls in the breaking of $3_c2_L2_R1_{B-L}$ down to the standard model gauge group. The cosmological relevance of these walls bounded by strings depends upon the presence of an inflationary epoch before the phase transition leading to the walls formation has taken place; see Sec. 3.6.1.

Embedded Defects

In these models with intermediate $3_c2_L2_R1_{B-L}$ symmetry, the breaking schemes are equivalent to

$$\mathrm{SO}(10) \stackrel{M_{\mathrm{GUT}}}{\rightarrow} \mathrm{G} \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B\text{–}L} \stackrel{M_{\mathrm{G}}}{\rightarrow} \mathrm{G} \times \mathrm{U}(1)_Y (\times Z_2) \stackrel{M_{\mathrm{Z}}}{\rightarrow} 3_c 1_Q (Z_2) \tag{3.50}$$

where $\mathrm{G} = \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L$. In direct analogy with electroweak strings [78], it is easy to see that embedded defects form during the second stage of symmetry breaking. They have a mass per unit length $\mu \sim 10^{30} - 10^{32}$ GeV^2. The stability conditions for these strings can be computed. If these strings are dynamically stable, they may generate density perturbations in the early universe and temperature anisotropy in the CBR.

Cosmic Strings

Consider the model where $3_c2_L2_R1_{B-L}$ breaks down to $3_c2_L1_YZ_2$. The first homotopy group $\pi_1(\frac{3_c2_L2_R1_{B-L}}{3_c2_L1_YZ_2}) = Z_2$ is non trivial which shows the formation of topological Z_2 strings. Since the Z_2 parity symmetry is kept unbroken down to low energy, the strings are topologically stable. They have a mass per unit length $\mu \sim 10^{30} - 10^{32}$ GeV^2. These strings will generate density perturbations in the early universe and temperature anisotropy in the CBR.

Solving the monopole problem

One can use an inflationary scenario as described in Sec. 3.4 to dilute the monopoles formed at M_{GUT}. Since the rank of $3_c2_L2_R1_{B-L}(Z_2^c)$ is four, the inflaton field will couple to a pair of $\mathbf{16}+\overline{\mathbf{16}}$ or $\mathbf{126}+\overline{\mathbf{126}}$ which will break $3_c2_L2_R1_{B-L}$, (see Sec. 3.4). Cosmic strings (if unbroken Z_2 symmetry at low energy) and/or domain walls (if unbroken Z_2^c symmetry at the intermediate scale) will form at the end of inflation. As shown in Sec. 3.6.1 the presence of this inflationary epoch between the two phase transitions at M_{GUT} and M_{G}, respectively, would make the walls dominate the energy density of the universe, (see Sec. 3.6.1). Now the unbroken Z_2 symmetry is necessary to preserve large values for the proton lifetime; hence, the only symmetry breaking pattern consistent with cosmology with intermediate $3_c2_L2_R1_{B-L}$ symmetry is

$$\mathrm{SO}(10) \stackrel{<45>+<54>}{\rightarrow} \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B\text{–}L} \tag{3.51}$$

$$\stackrel{<126>+<\overline{126}>}{\rightarrow} \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times Z_2 \tag{3.52}$$

$$\stackrel{<10>}{\rightarrow} \mathrm{SU}(3)_c \times \mathrm{U}(1)_Q \times Z_2 \tag{3.53}$$

where SO(10) is broken with a combination of a 45 dimensional Higgs representation and 54 dimensional one, $3_c2_L2_R1_{B-L}$ is broken with pair of $\mathbf{126}+\overline{\mathbf{126}}$ dimensional Higgs representation and $3_c2_L2_YZ_2$ is broken with a 10 Higgs multiplet.

3.7 Breaking via $3_c2_L1_R1_{B-L}$

We shall consider first the symmetry breaking with intermediate $3_c2_L1_R1_{B-L}$ symmetry without conserved matter parity at low energy :

$$\begin{aligned} \mathrm{SO}(10) &\stackrel{M_{\mathrm{GUT}}}{\to} \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_R \times \mathrm{U}(1)_{B\text{–}L} && (3.54)\\ &\stackrel{M_{\mathrm{G}}}{\to} \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y && (3.55)\\ &\stackrel{M_{\mathrm{Z}}}{\to} \mathrm{SU}(3)_c \times \mathrm{U}(1)_Q && (3.56)\end{aligned}$$

The first homotopy group $\pi_1(3_c2_L1_R1_{B\text{–}L}) = Z \oplus Z$ and therefore topological monopoles form during the first phase transition from supersymmetric SO(10) down to $3_c2_L1_R1_{B-L}$. These monopoles carry R and $B-L$, and have a mass $M_m \geq (10^{16} - 10^{17})$ GeV. Now $\pi_1(3_c2_L1_Y)$ and $\pi_1(3_c \times 1_Q)$ are both non trivial and hence, from an homotopy point of view, the monopoles are topologically stable. But as we are going to show below, some of these monopoles are indeed topologically stable, but some others will decay. During the second phase transition, the formation of strings is governed by the first homotopy group $\pi_1(\frac{3_c2_L1_R1_{B\text{–}L}}{3_c2_L1_Y}) = Z$, showing the formation of cosmic strings during the second phase transition. These are associated with the breaking of $\mathrm{U}(1)_R \times \mathrm{U}(1)_{B\text{–}L}$ down to $\mathrm{U}(1)_Y$ where the unbroken $\mathrm{U}(1)_R \times \mathrm{U}(1)_{B\text{–}L}$ symmetry in the first stage of symmetry breaking is responsible for the formation of monopoles. Now the weak hypercharge $\frac{Y}{2}$ is a linear combination of $B-L$ and R, $\frac{Y}{2} = (\frac{B\text{–}L}{2} + R)$. Therefore primordial monopoles with topological charge $\frac{B\text{–}L}{2} - R \neq 0$ get connected by the strings at the second stage of symmetry breaking. Some infinite and closed strings can also form. These cosmic strings are topologically unstable. They can break producing monopole-antimonopole pairs at the free ends. The monopole-antimonopole pairs connected by strings annihilate in less than a Hubble time and could produce the observed baryon asymmetry of the universe. Other monopoles formed during the first phase transition do not get connected by strings and remain stable down to low energy.

The monopole problem can be solved with an inflationary scenario as described in Sec. 3.4. Since the rank of $3_c2_L1_R1_{B-L}$ is five, the inflaton field will couple to the Higgs representation mediating the second phase transition associated with the breaking of $3_c2_L1_R1_{B-L}$. The monopoles can be pushed beyond the present horizon, and the monopole problem solved. Furthermore, since all the monopoles are inflated away, the string decay probability is negligible and the evolution of strings is identical to that of topologically stable strings. We therefore have a very interesting breaking scheme, where monopoles are created during a first transition, inflated away, and cosmic strings form at the end of inflation.

This model where $3_c2_L1_R1_{B\text{–}L}$ breaks down to the standard model without matter parity is in conflict with the actual data for proton lifetime. The solution to this problem is therefore that the intermediate subgroup break down to $3_c 2_L 1_Y Z_2$ as in model (3.13). In this case, topologically stable Z_2-strings will form during the second phase transition. They have a mass per unit length $\mu \sim (10^{30} - 10^{32})$ GeV^2. This interesting model with inflation and cosmic strings is studied in detail in the next chapter.

3.8 Breaking directly down to the standard model

Supersymmetric SO(10) can break directly down to the standard model as in model (3.7)

$$\mathrm{SO}(10) \stackrel{M_{\mathrm{GUT}}}{\to} \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \stackrel{M_Z}{\to} \mathrm{SU}(3)_c \times \mathrm{U}(1)_Q \tag{3.57}$$

or as in model (3.14)

$$\mathrm{SO}(10) \stackrel{M_{\mathrm{GUT}}}{\to} \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times Z_2 \stackrel{M_Z}{\to} \mathrm{SU}(3)_c \times \mathrm{U}(1)_Q \times Z_2 \tag{3.58}$$

with (3.58) or without (3.57) the Z_2 symmetry, subgroup of the Z_4 centre of SO(10), unbroken down to low energy. The latter plays the role of matter parity, giving large values for the proton lifetime and stabilising the LSP. The symmetry breaking occurs at $M_{\mathrm{GUT}} \simeq 2 \times 10^{16}$ GeV. The scenario without the unbroken Z_2 symmetry (3.57) is not, with the present data for proton decay, relevant phenomenologically.

In model (3.58), the Z_2 symmetry remains unbroken down to low energy, preserving large values for the proton lifetime. Furthermore, the first homotopy group $\pi_1(\frac{\mathrm{SO}(10)}{3_c 2_L 1_Y Z_2}) = \pi_0(3_c 2_L 1_Y Z_2)$ $= Z_2$ and therefore cosmic strings form when SO(10) breaks. They are associated with the unbroken Z_2 symmetry; since the latter remains unbroken down to low energy, the strings are topologically stable down to low energy. They have a mass per unit length $\mu \sim 10^{32}$ GeV^2. The latter could account for the density perturbations produced in the early universe which lead to galaxy formation and to temperature fluctuations in the CBR.

Again, due to the unbroken $\mathrm{U}(1)_Y$ symmetry, monopoles form at the grand unified phase transition. They carry Y topological charge and are topologically stable down to low energy. Their topological charge may change from Y to Q.

Since monopoles form in both models, the potential conflict with the standard big-bang cosmology is again not avoided. Nevertheless, in model (11), if the Higgs field leading to monopole production takes its vacuum expectation value (VEV) before inflation ends and the latter ends before the Higgs field leading to cosmic string formation acquires its VEV then we are left with a very attractive scenario.

Unfortunately, it does not seem possible to achieve this. If one attempts to inflate away the monopoles with a superpotential of the form given in Sec. 3.4, an intermediate scale is introduced. Thus, one is either left with the monopole problem in cosmology or loses the simplicity of this breaking scheme.

3.9 Conclusions

The aim of this chapter is to constrain supersymmetric SO(10) models which lead to the formation of topological defects through cosmological considerations. The main reason for considering supersymmetric versions of the grand unified gauge group SO(10), rather than nonsupersymmetric ones, is to predict the measured values of $\sin^2\theta_w$ and the gauge coupling constants merging in a single point at $\simeq 2\times 10^{16}$ GeV. Spontaneous symmetry breaking (SSB) patterns from supersymmetric SO(10) down to the standard model differ from nonsupersymmetric ones first in the scale of $B-L$ symmetry breaking and second in the ways of breaking from SO(10) down to the standard model. For nonsupersymmetric models the scale of $B-L$ breaking has to be anywhere between 10^{10} and $10^{13.5}$ GeV, whereas it is $\sim 10^{15}-10^{16}$ GeV in supersymmetric models. Furthermore, in the supersymmetric case, we can break directly down to the standard model without any intermediate breaking scale, and not more than one intermediate scale is expected. We have given a systematic analysis of topological defect formation and their cosmological implications in each model. We found that the rules for topological defect formation are not affected by the presence of supersymmetry, and since SO(10) is simply connected and the standard model gauge group involves an unbroken U(1) symmetry, all SSB patterns from supersymmetric SO(10) down to the standard model involve automatically the formation of topologically stable monopoles. In tables 3.1, 3.2, 3.3 and 3.4 we give a summary of all the defects formed in each model. In the models where Z_2-walls arise at the second phase transition, we have in fact hybrid defects. The walls are bounded by the Z_2-strings previously formed and are unstable. In order to solve the monopole problem, we propose an hybrid inflationary scenario [29, 30] which arises in supersymmetric SO(10) models without imposing any external symmetry and without imposing any external field (see Chap. 5). The inflationary scenario can cure the monopole problem, but then stabilises the Z_2 walls previously discussed. Hence these cases lead to another cosmological problem. Imposing also that the models satisfy the actual data on the proton lifetime, we found that there are only two SSB patterns consistent with cosmological considerations. Breaking directly to the standard model at first sight seems attractive. Unfortunately, one is unable to inflate away the monopoles without the introduction of an intermediate scale. The only breaking schemes consistent with cosmology correspond to the intermediate symmetry groups $3_C 2_L 2_R 1_{B-L}$, where SO(10) is broken with a combination of a 45 dimensional Higgs representation and a 54 dimensional one, and $3_C 2_L 1_R 1_{B-L}$. These intermediate symmetry groups must later break down to the standard model gauge group with unbroken matter parity; the symmetry breaking must be implemented with only Higgs fields in 'safe' representations [44], hence the rank of the group must be lowered with a pair of Higgs fields in the $(\mathbf{126}+\overline{\mathbf{126}})$ dimensional representation, and the standard model gauge group broken with two **10**-dimensional ones. The model with intermediate $3_C 2_L 1_R 1_{B-L}$, inflation, and cosmic strings, is studied in detail in the next chapter. In supergravity SO(10) models, the breaking of SO(10) via flipped SU(5) is also possible.

G	SO(10) $\to G$	$G \to 3_c 2_L 1_Y$	Cosmological problems
$SU(5) \times U(1)_X$	monopoles-1	monopoles-2 + strings	monopoles-2 + proton lifetime (Z_2 broken)
SU(5)	no defects	monopoles	monopoles + proton lifetime (Z_2 broken)
$SU(5) \times \widetilde{U(1)}$	monopoles	embedded strings	proton lifetime (Z_2 broken)
$4_c 2_L 2_R$	monopoles-1	monopoles-2	monopoles-2 + proton lifetime (Z_2 broken)
$4_c 2_L 2_R Z_2^c$	monopoles-1 + Z_2-strings	monopoles-2 + Z_2-walls	Z_2-walls and monopoles-2 + proton lifetime (Z_2 broken)
$3_c 2_L 2_R 1_{B-L}$	monopoles	embedded strings	proton lifetime (Z_2 broken)
$3_c 2_L 2_R 1_{B-L} Z_2^c$	monopoles + Z_2-strings	embedded strings + Z_2-walls	Z_2-walls + proton lifetime (Z_2 broken)
$3_c 2_L 1_R 1_{B-L}$	monopoles	strings	proton lifetime (Z_2 broken)

Table 3.1 : Formation of topological defects in the possible symmetry breaking patterns from supersymmetric SO(10) down to the standard model with broken matter parity. These models are inconsistent with proton lifetime measurements. The table also shows the relevant cosmological problems associated with each symmetry breaking pattern, when occuring within a hybrid inflationary scenario. From a topological defect point of view, models with intermediate $SU(5) \times \widetilde{U(1)}$, $3_c 2_L 2_R 1_{B-L}$ and $3_c 2_L 1_R 1_{B-L}$ symmetry groups are compatible with observations. The model with an intermediate $SU(5) \times \widetilde{U(1)}$ symmetry is only possible in supergravity SO(10) models.

G	SO(10) $\rightarrow G$	$G \rightarrow 3_c 2_L 1_Y Z_2$	Cosmological problems
SU(5) $\times$ U(1)$_X$	monopoles-1	monopoles + Z_2-strings	monopoles-2
SU(5) $\times Z_2$	Z_2-strings	monopoles-2	monopoles-2
SU(5) $\times \widetilde{\mathrm{U}(1)}$	monopoles	Z_2-strings	no problem, monopoles inflated away
$4_c 2_L 2_R$	monopoles-1	monopoles-2 + Z_2-strings	monopoles-2
$4_c 2_L 2_R Z_2^c$	monopoles-1 + Z_2-strings	monopoles-2 + Z_2-strings + Z_2-walls	monopoles-2 + Z_2-walls
$3_c 2_L 2_R 1_{B-L}$	monopoles	embedded strings + Z_2-strings	no problem, monopoles inflated away
$3_c 2_L 2_R 1_{B-L} Z_2^c$	monopoles + Z_2-strings	embedded strings + Z_2-strings + Z_2-walls	Z_2-walls
$3_c 2_L 1_R 1_{B-L}$	monopoles	Z_2-strings	no problem, monopoles inflated away

Table 3.2 : Formation of topological defects in the possible symmetry breaking patterns from supersymmetric SO(10) down to the standard model with unbroken matter parity. These models are consistent with proton life time measurements and can provide a superheavy Majorana mass to the right-handed neutrinos. The table also shows the relevant cosmological problems associated with each symmetry breaking pattern, when occurring within a hybrid inflationary scenario. The models with intermediate SU(5) $\times \widetilde{\mathrm{U}(1)}$, $3_c 2_L 2_R 1_{B-L}$ and $3_c 2_L 1_R 1_{B-L}$ symmetry groups are consistent with observations. The model with intermediate SU(5) $\times \widetilde{\mathrm{U}(1)}$ symmetry is only possible in supergravity SO(10) models.

SO(10) $\rightarrow 3_c 2_L 1_Y$	Cosmological problems
monopoles-2	monopoles-2 + proton life-time (Z_2 broken)

Table 3.3 : Formation of topological defects in models where supersymmetric SO(10) breaks directly down to the MSSM with broken matter parity. The table also shows the relevant cosmological problems associated with the symmetry breaking pattern, when occurring within a hybrid inflationary scenario. These models are inconsistent with observations.

SO(10) $\rightarrow 3_c 2_L 1_Y Z_2$	Cosmological problems
monopoles-2 + Z_2-strings	monopoles-2

Table 3.4 : Formation of topological defects in models where supersymmetric SO(10) breaks directly down to the MSSM with unbroken matter parity. The table also shows the relevant cosmological problems associated with the symmetry breaking pattern, when occurring within a hybrid inflationary scenario. These models are inconsistent with observations.

Chapter 4

Supersymmetric SO(10) model with inflation and cosmic strings

4.1 Introduction

In the previous chapter, we have constrained supersymmetric SO(10) models using both cosmological and particle physics arguments. We have in particular studied the formation of topological defects in all possible symmetry breaking patterns from supersymmetric SO(10) down to the standard model, considering no more than one intermediate symmetry breaking scale. Recall that domain walls and monopoles are in conflict with the standard cosmology whereas cosmic strings may have interesting cosmological consequences. Since SO(10) is simply connected and the standard model gauge group involves an unbroken U(1) symmetry, which remains unbroken down to low energy, all symmetry breaking patterns from supersymmetric SO(10) down to the standard model automatically lead to the formation of topologically stable monopoles. All supersymmetric SO(10) models are therefore cosmologically irrelevant without invoking some mechanism for the removal of the monopoles, such as an inflationary scenario. The conclusion in Chap. 3 is that there are only two possibilities for breaking SO(10) down to the standard model which are consistent with observations. SO(10) can break via $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$, here SO(10) must be broken with a combination of a 45-dimensional Higgs representation and a 54-dimensional one, and via $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_R \times \mathrm{U}(1)_{B-L}$. In these models, the intermediate symmetry group must be broken down to the standard model gauge group with unbroken matter parity, $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times Z_2$. In supergravity SO(10) models, the breaking of SO(10) via flipped SU(5) is also possible.

In this chapter, we study a supersymmetric SO(10) model involving an intermediate $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_R \times \mathrm{U}(1)_{B-L}$ symmetry. The resultant cosmological model is compatible with observations.

In Sec. 4.2, we describe a hybrid inflationary scenario, already introduced in Sec. 3.4, and we argue that this type of inflationary scenario occurs naturally in global supersymmetric SO(10) models. Neither any external field nor any external symmetry has to be imposed. We give the general form of the potential which, in supersymmetric SO(10) models, leads to the required spontaneous symmetry breaking pattern and gives rise to such a period of inflation.

In the next sections, we construct a specific supersymmetric SO(10) model, as mentioned above. The latter aims to be consistent with observations. In Sec. 4.3 we study the symmetry breaking pattern. We conclude on the proton lifetime and on a hot dark-matter candidate provided by the model. Using homotopy theory, we find topological defects which form according to the Kibble mechanism [27].

In Sec. 4.4, we explain how to implement the symmetry breaking pattern which solves the doublet-triplet splitting and includes the inflationary scenario described in Sec. 4.2. We write down the superpotential and find its global minimum with corresponding Higgs VEVs.

In Sec. 4.5, we evaluate the dynamics of the symmetry breaking and inflationary scenario, studying the scalar potential. It is shown that the monopole problem may be solved and that cosmic strings form at the end of inflation.

In Sec. 4.6, we give general properties of the strings formed at the end of inflation. In particular, we study the possibility that the strings may be superconducting.

In Sec. 4.7, we estimate the observational consequences. The temperature fluctuations in the CBR due to the mixed inflation-cosmic strings scenario are evaluated. Using the temperature fluctuations measured by COBE we find values for the scalar coupling constant, the scale at which the strings formed and the strings mass per unit length. We specify the dark-matter present in the model and give a qualitative discussion of the large-scale structure formation scenario in this model.

We finally conclude in Sec. 4.8.

We shall use the notations given in Eqs. (3.15)-(3.19).

4.2 Inflation in supersymmetric SO(10) models

In this section, we argue that false vacuum hybrid inflation, with a superpotential in the inflaton sector similar to that studied in Refs. [29, 30], is a natural mechanism for inflation in global supersymmetric SO(10) models. Neither any external field nor any external symmetry has to be imposed, it can just be a consequence of the theory.

The first thing to note in SO(10) models, is that the rank of SO(10) is greater than one unit from the rank of the standard model gauge group. The rank of SO(10) is five, whereas the rank of the standard model gauge group $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y (\times Z_2)$ is four. In other words, SO(10) has an additional U(1) symmetry, named $\mathrm{U}(1)_{B-L}$, compared to the standard model gauge group. Therefore the rank of the group must be lowered by one unit at some stage of the symmetry breaking pattern, i.e., $\mathrm{U}(1)_{B-L}$ must be broken. This can be done using a pair of $\mathbf{16}+\overline{\mathbf{16}}$ Higgs representations or by a pair of $\mathbf{126}+\overline{\mathbf{126}}$ representations. If a $\mathbf{16}+\overline{\mathbf{16}}$ pair of Higgs fields are used, then the Z_2 symmetry, subgroup of both the Z_4 centre of SO(10) and of $\mathrm{U}(1)_{B-L}$ is broken. On the other hand, if a $\mathbf{126}+\overline{\mathbf{126}}$ pair of Higgs fields are used, then the Z_2 symmetry can be kept unbroken down to low energy if only safe representations [44] are used to implement the full symmetry breaking pattern, such as the 10, the 45, the 54 or the 210-dimensional representations. If a $\mathbf{126}+\overline{\mathbf{126}}$ pair of Higgs fields are used, the right-handed neutrino can get a superheavy Majorana mass, and the solar neutrino problem can be solved via the MSW mechanism [40].

In order to force the VEVs of the $\mathbf{16}+\overline{\mathbf{16}}$ or $\mathbf{126}+\overline{\mathbf{126}}$ pair of Higgs fields, needed to lower the rank of the group, to be the order of the GUT scale, we can use a scalar field $\mathcal{S}$ singlet under SO(10). The superpotential can be written as follows,

$$W = \alpha \mathcal{S}\overline{\Phi}\Phi - \mu^2 \mathcal{S} \tag{4.1}$$

where $\Phi + \overline{\Phi}$ stand for a $\mathbf{16}+\overline{\mathbf{16}}$ or a $\mathbf{126}+\overline{\mathbf{126}}$ pair of Higgs fields, and the field $\mathcal{S}$ is a scalar field singlet under SO(10). The constants α and μ are assumed to be both positive and must satisfy $\frac{\mu}{\sqrt{\alpha}} \sim (10^{15} - 10^{16})$ GeV.

It is easy to see that the superpotential given in Eq. (4.1), used to break the rank of the group by one unit, is the same superpotential used by Dvali *et al.*[29, 77] to implement a false vacuum hybrid inflationary scenario, identifying the scalar field $\mathcal{S}$ with the inflaton field. Hence, as shown below, in supersymmetric SO(10) models, the superpotential used to break $\mathrm{U}(1)_{B-L}$ can also lead to a period of inflation. Inflation is then just a consequence of the theory. In order to understand the symmetry breaking and the inflationary dynamics, we can study the scalar potential. From Eq. (1.60), the latter is given by (keeping the same notation for the bosonic component of the superfields as for the superfields):

$$V = |F_S|^2 + |F_\Phi|^2 + |F_{\overline{\Phi}}|^2 + D - \text{terms} \tag{4.2}$$

where the F terms are such that $F_{\Psi_i} = |\frac{\partial W}{\partial \Psi_i}|$, for $\Psi_i = \mathcal{S}, \Phi$ and $\overline{\Phi}$. The D-terms vanish if $|\Phi| = |\overline{\Phi}|$. Therefore the Higgs potential

$$V = \alpha^2 |\mathcal{S}\overline{\Phi}|^2 + \alpha^2 |\mathcal{S}\Phi|^2 + |\alpha\overline{\Phi}\Phi - \mu^2|^2 \,. \tag{4.3}$$

It is minimised for $\arg(\Phi) + \arg(\overline{\Phi}) = 0$, $(\alpha > 0)$, and it is independent of $\arg(\mathcal{S}) + \arg(\Phi)$ and $\arg(\mathcal{S}) + \arg(\overline{\Phi})$. Thus we can rewrite the scalar potential with the new fields which minimise the potential, keeping the same notation for the old and new fields,

$$V = 4\alpha^2 |\mathcal{S}|^2 |\Phi|^2 + (\alpha|\Phi|^2 - \mu^2)^2 \,. \tag{4.4}$$

The potential has a unique supersymmetric minimum corresponding to $<|\Phi|>=<|\overline{\Phi}|>= \frac{\mu}{\sqrt{\alpha}}$ and $\mathcal{S} = 0$. The potential has also a local minimum corresponding $\mathcal{S} > \frac{\mu}{\sqrt{\alpha}}$ and $<|\Phi|>= <|\overline{\Phi}|>= 0$. We identify the scalar field $\mathcal{S}$ with the inflaton field and we assume chaotic initial conditions. All the fields are thus supposed to take initial values the order of the Planck scale, and hence the initial value of the inflaton field $\mathcal{S} \gg \frac{\mu}{\sqrt{\alpha}}$. Since the potential is flat in the $\mathcal{S}$ direction, we can minimise it at a fixed value of $\mathcal{S}$. The Φ and $\overline{\Phi}$ fields roll down their local minimum corresponding to $<|\Phi|>=<|\overline{\Phi}|>=0$. The vacuum energy density is then dominated by a non vanishing $F_{\mathcal{S}}$ term, $|F_{\mathcal{S}}| = \mu^2$. $F_{\mathcal{S}} \neq 0$ implies that supersymmetry is broken. The inflationary epoch takes place as the inflaton field slowly rolls down the potential. Quantum corrections to the effective potential will help the fields to slowly roll down their global minimum [29]. At the end of inflation, the phase transition mediated by the Φ and $\overline{\Phi}$ fields takes place.

Now, in order to break SO(10) down to the standard model gauge group, we need more than a $\mathbf{16} + \overline{\mathbf{16}}$ or a $\mathbf{126} + \overline{\mathbf{126}}$ pair of Higgs fields. We need Higgs in other representations, like the 45, 54 or 210-dimensional representations if the Z_2 parity is to be kept unbroken down to low energy, as required from proton lifetime measurements. Thus the full superpotential needed to break SO(10) down to the standard model must, apart of Eq. (4.1), contains terms involving the other Higgs needed to implement the symmetry breaking. Due to the nonrenormalisation theorem in supersymmetric theories, we can write down the full superpotential which can implement the desired symmetry breaking pattern, just adding to Eq. (4.1) terms mixing the other Higgs needed to implement the symmetry breaking pattern. There can be no mixing between the latter Higgs and the pair of Higgs used to break $\mathrm{U}(1)_{B-L}$ (see Sec. 4.4 for example) and the superpotential can be written as follows :

$$W_{\mathrm{tot}} = W(\mathcal{S}, \Phi, \overline{\Phi}) + W_1(H_1, H_2, ..) \tag{4.5}$$

where $\mathcal{S}$ is a scalar field singlet under SO(10) identified with the inflaton field, the Φ and $\overline{\Phi}$ fields are the Higgs fields used to break $\mathrm{U}(1)_{B-L}$ and the H_i fields, $i = 1, .., m$, are the m other Higgs fields needed to implement the full symmetry breaking pattern from SO(10) down to the standard model gauge group. W is given by Eq. (4.1) and $W + W_1$ has a global supersymmetric minimum such that the SO(10) symmetry group is broken down to the standard model gauge group. The scalar potential is then given by

$$V_{\mathrm{tot}} = V(\mathcal{S}, \Phi, \overline{\Phi}) + V_1(H_i)\ . \tag{4.6}$$

V is given by Eq. (4.4) and $V + V_1$ has a global minimum such that the SO(10) symmetry is broken down to the standard model gauge group. The evolution of the fields is then as follows. The fields take random initial values, just subject to the constraint that the energy density is at the Planck scale. The inflaton field is distinguished from the other fields from the fact that the GUT potential is flat in its direction; the potential can be minimised for fixed $\mathcal{S}$. Chaotic initial conditions imply that the initial value of the inflaton field is greater than $\frac{\mu}{\sqrt{\alpha}}$. Therefore, the non-inflaton fields will roll very quickly down to their global (or local) minimum, at approximately a fixed value for the inflaton, $<|H_i|>\neq 0$, for $i = 1, ..., n$, $<|H_j|>= 0$, for $j = n+1, ..., m$, and $<|\Phi|>=<|\overline{\Phi}|>= 0$; a first symmetry breaking, implemented by the n Higgs fields H acquiring VEV, takes place, SO(10) breaks down to an intermediate symmetry group G. Then inflation occurs as the inflaton rolls slowly down the potential. The symmetry breaking implemented with the $\Phi + \overline{\Phi}$ fields occurs at the end of inflation, and the the intermediate symmetry group G breaks down to the standard model gauge group.

In the scenario described above, the rank of the intermediate symmetry group G is equal to the rank of SO(10), which is five, and hence involves an unbroken $\mathrm{U}(1)_{B-L}$ symmetry. If the rank of the intermediate symmetry group were that of the standard model gauge group, that is if $\mathrm{U}(1)_{B-L}$ were broken at the first stage of the symmetry breaking, the inflationary scenario would unable to solve the monopole problem, since the later would form at the end of inflation or once inflation completed. Finally, in models where supersymmetric SO(10) is broken directly down to the standard model gauge group, such hybrid inflationary scenarios cannot cure the monopole problem.

We conclude that if inflation has to occur during the evolution of the universe described by a spontaneous symmetric breaking pattern from the supersymmetric grand unified gauge group SO(10) down to the minimal supersymmetric standard model, it can thus just be a consequence of the theory. No external field and no external symmetry has to be imposed. One can use the superpotential given in Eq. (4.1) to lower the rank of the group by one unit and then identify the scalar field $\mathcal{S}$, singlet under SO(10), with the inflaton field. A false vacuum hybrid inflationary scenario will be implemented. It emerges from the theory.

4.3 The supersymmetric SO(10) model and the standard cosmology

We now construct a supersymmetric SO(10) model which aims to agree with observations. SO(10) is broken down to the standard model gauge group with unbroken matter parity $3_c 2_L 1_Y Z_2$, via the intermediate symmetry group $3_c 2_L 1_R 1_{B-L}$. We study the symmetry breaking pattern of the model and deduce general impacts of the model on observations. We look for topological defects formation.

The model initially assumes that the symmetries between particles, forces and particles, are described by a supersymmetric SO(10) theory. The SO(10) symmetry is then broken down to the standard model gauge group via $3_c 2_L 1_R 1_{B-L}$,

$$\mathrm{SO}(10) \stackrel{M_{\mathrm{GUT}}}{\to} 3_c 2_L 1_R 1_{B-L} \stackrel{M_{\mathrm{G}}}{\to} 3_c 2_L 1_Y Z_2 \stackrel{M_{\mathrm{Z}}}{\to} 3_c 1_Q Z_2, \tag{4.7}$$

$M_{\mathrm{GUT}} \sim 10^{16}$ GeV , $M_{\mathrm{G}} \sim M_{\mathrm{GUT}}$ with $M_{\mathrm{G}} \leq M_{\mathrm{GUT}}$ and $M_{\mathrm{Z}} \simeq 100$ GeV, and supersymmetry is broken at $M_{\mathrm{s}} \sim 10^3$ GeV. Recall that the Z_2 symmetry, which appears at the second stage of the symmetry breaking in (4.7), is the discrete$\{1, -1\}$ symmetry, subgroup of both the Z_4 centre of SO(10) and of $\mathrm{U}(1)_{B-L}$ subgroup of SO(10). The Z_2 symmetry acts as R-parity. It preserves large values for the proton lifetime and stabilises the LSP; it is thus necessary that this Z_2 symmetry be kept unbroken down to low energies.

In Sec. 3.7, we discussed the formation of topological defect in the symmetry breaking pattern given in Eq. (4.7). We now summarise our results. Monopoles form according to the Kibble mechanism during the first phase transition at M_{GUT} when SO(10) breaks down to $3_c 2_L 1_R 1_{B-L}$. Half of these monopoles are topologically stable down to low energies. During the second phase transition, when the $3_c2_L1_R1_{B-L}$ symmetry group breaks down to $3_c2_L1_YZ_2$ at M_{G}, cosmic strings form. The strings connect half of the monopole-antimonopole pairs formed earlier. Some closed strings can also form. The strings can break with monopole-antimonopole pair nucleation. The monopoles get attracted to each other and the whole system of strings disappears. Nevertheless, the other half of the monopoles, which do not get connected by strings, remain topologically stable, and are thus in conflict with the standard cosmology.

Now the rank of $3_c2_L1_R1_{B-L}$ is equal to five, as the rank of SO(10), and is therefore greater than the rank of $3_c2_L1_YZ_2$ from one unit. Thus we can couple the inflaton field with the Higgs field mediating the breaking of $3_c2_L1_R1_{B-L}$ down to $3_c2_L1_YZ_2$, see Sec. 4.2, and the monopole problem can be cured. If the monopoles are pushed away before the phase transition leading to the strings formation takes place, then the evolution of the string network is quite different than previously said. It is that of topologically stable cosmic strings.

4.4 Model building

4.4.1 Ingredients

In this section, we explain how to implement the symmetry breaking pattern given in Eq. (4.7). The model solves the doublet-triplet splitting and includes an inflationary scenario as described in Sec. 4.2.

In order to implement the symmetry breaking pattern given in Eq. (4.7) and in order to preserve the Z_2 symmetry unbroken down to low energy, see Eq. (4.7), we must only use Higgs fields in 'safe' representations [44], such as the adjoint 45, the 54, the **126** or the 210-dimensional representations.

In order to implement the first stage of the symmetry breaking, we could use only one Higgs in the 210-dimensional representation; unfortunately the model would then not solve the doublet-triplet splitting problem. The latter can be easily solved using the Dimopoulos-Wilczek mechanism [63], using two Higgs, one in the adjoint 45-dimensional representation and one in the 54-dimensional one. The VEV of the adjoint 45, which we call A_{45}, which implements the Dimopoulos-Wilczek mechanism is in the $B-L$ direction, and breaks SO(10) down to $3_c2_L2_R1_{B-L}$. The Higgs in the 54 dimensional representation, which we call S_{54}, breaks SO(10) down to $4_c2_L2_R$. Altogether the SO(10) symmetry is broken down to $3_c2_L2_R1_{B-L}$.

We want to break SO(10) directly down to $3_c2_L1_R1_{B-L}$, we therefore need more Higgs. We use another 54, which we call S'_{54}, and another 45, which we call A'_{45}, in the T_{3R} direction. The latter breaks SO(10) down to $4_c2_L1_R$. S'_{54} and A'_{45} break together SO(10) down to $4_c2_L1_R$.

The role of S_{54} and S'_{54} is to force A_{45} and A'_{45} into $B-L$ and T_{3R} directions. SO(10) breaks down to $3_c2_L1_R1_{B-L}$ with A_{45}, S_{54}, A'_{45} and S'_{54} acquiring VEVs, and as mentioned in Sec. 4.3, topologically stable monopoles form.

As discussed in Sec. 4.3, since the rank of $3_c2_L1_R1_{B-L}$ is equal to the rank of SO(10) which is five whereas the rank of $3_c2_L1_YZ_2$ is four, we can therefore implement a false vacuum hybrid inflationary scenario as described in Sec. 4.2, if we couple the inflaton field to the Higgs field used to break the intermediate symmetry gauge group $3_c 2_L 1_R 1_{B-L}$. The monopole problem can be solved and cosmic strings can form at the end of inflation when the $3_c 2_L 1_R 1_{B-L}$ symmetry group breaks down to the standard model gauge group with unbroken matter parity, $3_c 2_L 1_Y Z_2$.

To break $3_c 2_L 1_R 1_{B-L}$, we use a $\mathbf{126} + \overline{\mathbf{126}}$ pair of Higgs fields, which we call Φ_{126} and $\overline{\Phi}_{126}$. The latter are safe representations [44] and therefore keeps the Z_2 symmetry unbroken. A $\mathbf{16} + \overline{\mathbf{16}}$ pair of Higgs fields usually used for the same purpose would break the Z_2 symmetry. The VEV of the $\mathbf{126}$ and $\overline{\mathbf{126}}$ are in the X direction, the U(1) symmetry of SO(10) which commutes with SU(5). They break SO(10) down to SU(5) $\times$ Z_2. All together, i.e., with A_{45}, S_{54}, A'_{45}, S'_{54}, Φ_{126}, and $\overline{\Phi}_{126}$ acquiring VEVs, the SO(10) symmetry group is broken down to $3_c 2_L 1_Y Z_2$.

The symmetry breaking of the standard model is then achieved using two Higgs in the 10-dimensional representation of SO(10), H_{10} and H'_{10}.

To summarise, the symmetry breaking is implemented as follows:

$$\mathrm{SO}(10) \stackrel{<A_{45}><S_{54}><A'_{45}><S'_{54}>}{\rightarrow} 3_c 2_L 1_R 1_{B-L} \stackrel{<\Phi_{126}><\overline{\phi_{126}}>}{\rightarrow} 3_c 2_L 1_Y Z_2 \stackrel{<H_{10}><H'_{10}>}{\rightarrow} 3_c 1_Q Z_2 \,. \quad (4.8)$$

4.4.2 The superpotential

We now write down the superpotential involving the above mentioned fields. A consequence of the superpotential is the symmetry breaking pattern given in Eq. (4.7), which involves an inflationary sector.

As discussed above, our model involves four sectors. The first sector implements the doublet-triplet splitting and involves A_{45}, with VEV in the $\mathrm{U}(1)_{B-L}$ direction. It also involves S_{54} and two Higgs 10-plets, H and H'. The superpotential in the first sector is given by $W_1 + W_2$, with, dropping the subscripts,

$$W_1 = m_A A^2 + m_S S^2 + \lambda_S S^3 + \lambda_A A^2 S \quad (4.9)$$

and

$$W_2 = HAH' + m_{H'} {H'}^2 \,. \quad (4.10)$$

The Higgs potential V_1 associated with the superpotential W_1 has a global minimum such that the SO(10) symmetry group is broken down to $3_c 2_L 2_R 1_{B-L}$, with A_{45} and S_{54} acquiring VEVs. W_2 implements the doublet-triplet splitting; H and H' break $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ down to $\mathrm{U}(1)_Q$.

The second sector involves A'_{45}, with VEV in the T_{3R} direction, and S'_{54}. The superpotential in the second sector is given by

$$W_3 = m_{A'} {A'}^2 + m_{S'} {S'}^2 + \lambda_{S'} {S'}^3 + \lambda_{A'} {A'}^2 S' \,. \quad (4.11)$$

The associated Higgs potential V_3 has a global minimum such that the SO(10) symmetry group is broken down to $3_c 2_L 2_R 1_{B-L}$, with A'_{45} and S'_{54} acquiring VEVs.

The Higgs potential $V_1 + V_2 + V_3$ has a global minimum such that the SO(10) symmetry is broken down to $3_c 2_L 1_R 1_{B-L}$, with A_{45}, S_{54}, A'_{45} and S'_{54} acquiring VEVs.

The third sector involves Φ_{126} and $\overline{\Phi}_{126}$, and breaks SO(10) down to SU(5) $\times Z_2$. In order to force the Φ_{126} and $\overline{\Phi}_{126}$ fields to get their VEVs the order of the GUT scale, we use a scalar field $\mathcal{S}$ singlet under SO(10). The superpotential is of the form, dropping the subscripts,

$$W_4 = \alpha \mathcal{S} \overline{\Phi} \Phi - \mu^2 \mathcal{S} \; . \tag{4.12}$$

α and μ are both positive and we must have $\frac{\mu}{\sqrt{\alpha}} = M_{\mathrm{G}}$, with $M_{\mathrm{G}} \simeq 10^{15} - 10^{16}$ GeV for the unification of the gauge coupling constants. Identifying the scalar field $\mathcal{S}$ with the inflaton field, W_4 leads to a false vacuum hybrid inflationary scenario, as described in Sec. 4.2.

The Higgs potential $V_1 + V_2 + V_3 + V_4$ has a global minimum such that the $3_c 2_L 1_R 1_{B-L}$ symmetry group is broken down to the standard model gauge group with unbroken matter parity, $3_c 2_L 1_Y Z_2$, with A_{45}, S_{54}, A'_{45}, S'_{54}, Φ_{126} and $\overline{\Phi}_{126}$ acquiring VEVs.

The full superpotential $W_{\mathrm{tot}} = W_1 + W_2 + W_3 + W_4$ does not involve terms mixing A'_{45} and S_{54}, S'_{54} and A_{45} etc... . In other words the three sectors are independent. Thanks to the nonrenormalisable theorem, we are not obliged to write down these terms, and it is not compatible with any extra discrete symmetry [80], therefore we do not have to fear any domain wall formation when the symmetry breaks. Nevertheless, in order to avoid any undesirable massless Goldstone Bosons, the three sectors have to be related. The latter can be done introducing a third adjoint $A"_{45}$, and adding a term of the form $AA'A''$ to the superpotential [80]. The latter would neither affect the symmetry breaking pattern, nor the inflationary scenario discussed below. The full superpotential of the model is,

$$\begin{aligned} W_{\mathrm{tot}} \quad = \quad & m_A A^2 + m_S S^2 + \lambda_S S^3 + \lambda_A A^2 S + HAH' + m_{H'} {H'}^2 \\ & + m_{A'} {A'}^2 + m_{S'} {S'}^2 + \lambda_{S'} {S'}^3 + \lambda_{A'} {A'}^2 S' \\ & + \alpha \mathcal{S} \overline{\Phi} \Phi - \mu^2 \mathcal{S} \; . \end{aligned} \tag{4.13}$$

In Eq. (4.13), A^2 really means Tr(A^2), $A^2 S$ really means Tr$(A^2 S)$, etc. The superpotential given in Eq. (4.13) leads to the desired pattern of symmetry breaking and the VEVs of A_{45}, S_{54}, A'_{45}, S'_{54}, Φ_{126} and $\overline{\Phi}_{126}$ are given as follows (see Appendix C). The adjoint $< A_{45} >$ is in the $B - L$ direction,

$$< A_{45} >= J \otimes \mathrm{diag}(a, a, a, 0, 0) \tag{4.14}$$

where $J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ and $a \sim M_{\mathrm{GUT}}$. $< S_{54} >$ is a traceless symmetric tensor given by,

$$< S_{54} >= I \otimes \mathrm{diag}(x, x, x, -\frac{3}{2}x, -\frac{3}{2}x) \tag{4.15}$$

where I is the unitary 2×2 matrix and $x = -\frac{m_A}{2\lambda_A}$. $< A'_{45} >$ is in the T_{3R} direction,

$$< A'_{45} >= J \otimes \mathrm{diag}(0, 0, 0, a', a'). \tag{4.16}$$

where $a' \sim M_{\rm GUT}$. S'_{54} is a traceless antisymmetric tensor,

$$< S'_{54} >= I \otimes diag(x,' x', x', -\frac{3}{2}x', -\frac{3}{2}x') \tag{4.17}$$

where $x' = \frac{2m_{A'}}{3\lambda_{A'}}$. The only component of the **126** which acquires VEV is in the direction of the right-handed neutrino (it is the component which transforms as a singlet under SU(5))

$$< |\Phi_{126}| >_{\nu^c\nu^c} = < |\overline{\Phi}_{126}| >_{\overline{\nu^c\nu^c}} = d\,. \tag{4.18}$$

Finally, we give VEV to the components of the 10 dimensional Higgs fields which correspond to the usual Higgs doublets. We do not use these, since we are interested in higher energies, where inflation and the GUT phase transitions take place.

With the VEVs above, if $< \mathcal{S} >= 0$ and $d = \frac{\mu}{\sqrt{\alpha}}$, the Higgs potential has a global minimum such that the SO(10) symmetry is broken down to the standard model gauge group with unbroken matter parity $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times Z_2$, and supersymmetry is unbroken (see App. C).

4.5 The inflationary epoch

In this section we evaluate the details of the symmetry breaking pattern and of the inflationary scenario. We write down the scalar potential and find values of the scalar coupling constant and the mass scales $M_{\rm G}$ and $M_{\rm GUT}$ for which the inflationary scenario is successful.

We are interested in the dynamics of the symmetry breaking pattern and how the inflationary scenario fits in the symmetry breaking pattern. We therefore need to study the scalar potential. In order fully to understand the dynamics of the model, one would need to use finite temperature field theory. Nevertheless, the study of the scalar potential derived from the superpotential given in Eq. (4.13) leads to a good understanding of the field evolution. We are mainly interested in what is happening above the electroweak scale, and hence we do not take into account the 10-dimensional Higgs multiplets H and H' which only break the standard model gauge group. The scalar potential is then given by

$$\begin{aligned} V &= (2m_A A + 2\lambda_A A S)^2 + (2m_S S + 3\lambda_S S^2 + \lambda_A A^2)^2 \\ &\quad + (2m_{A'} A' + 2\lambda_{A'} A' S')^2 + (2m_{S'} S' + 3\lambda_{S'} S'^2 + \lambda_{A'} A'^2)^2 \\ &\quad + \alpha^2 |\mathcal{S}\overline{\Phi}|^2 + \alpha^2 |\mathcal{S}\Phi|^2 + |\alpha\overline{\Phi}\Phi - \mu^2|^2. \end{aligned} \tag{4.19}$$

We remind the reader that A and A' are two Higgs in the 45-dimensional representation of SO(10) with VEV in the $B-L$ and T_{3R} directions, respectively. S and S' are two Higgs in the 54-dimensional representation of SO(10). Φ and $\overline{\Phi}$ are two Higgs in the **126** and $\overline{\mathbf{126}}$ representations, with VEVs in the right-handed neutrino direction. The scalar field $\mathcal{S}$ is a singlet under SO(10) and is identified with the inflaton field. α and μ are both positive constants which must satisfy the relation $\frac{\mu}{\sqrt{\alpha}} = M_{\rm G}$. The potential is minimised for $\arg(\Phi)+\arg(\overline{\Phi}) = 0$, $(\alpha > 0)$, and it is independent of $\arg(\mathcal{S})+\arg(\Phi)$ and $\arg(\mathcal{S})+\arg(\overline{\Phi})$. We rewrite the potential with the new fields which minimise the potential, keeping the same notation for the old and new fields. The Higgs potential becomes

$$\begin{aligned} V &= (2m_A A + 2\lambda_A AS)^2 + (2m_S S + 3\lambda_S S^2 + \lambda_A A^2)^2 \\ &+ (2m_{A'} A' + 2\lambda_{A'} A'S')^2 + (2m_{S'} S' + 3\lambda_{S'} S'^2 + \lambda_{A'} A'^2)^2 \\ &+ 4\alpha^2 |\mathcal{S}|^2 |\Phi|^2 + (\alpha|\Phi|^2 - \mu^2)^2 + \frac{1}{2} m^2 |\mathcal{S}|^2, \end{aligned} \tag{4.20}$$

where we have also introduced a soft supersymmetry breaking term for $\mathcal{S}$, and $m \sim 10^3$ GeV.

The scalar potential is flat in the $\mathcal{S}$ direction; we thus identify the scalar field $\mathcal{S}$ with the inflaton field. We suppose chaotic initial conditions; that is we suppose that all the fields have initial values of the order of the Planck scale. We then minimise the superpotential for fixed $\mathcal{S}$. We easily find that for $|\mathcal{S}| > \frac{\mu}{\sqrt{\alpha}} = s_c$, (recall that $\mu, \alpha > 0$), there is a local minimum corresponding to $|\Phi| = |\overline{\Phi}| = 0$, and A, A', S and S' taking values as given above in equations (4.14), (4.15), (4.16), and (4.17). Since all the fields are assumed to take initial values of the order of the Planck scale, the inflaton field has an initial value greater than $\frac{\mu}{\sqrt{\alpha}}$. Then, because the potential is flat in the inflaton direction, the fields Φ, $\overline{\Phi}$, A, A', S and S' settle quickly to the local minimum corresponding to $< S >$, $< A >$, $< S' >$ and $< A' >$ as in equations (4.14), (4.15), (4.16) and (4.17) respectively, and $<|\Phi|>=<|\overline{\Phi}|>= 0$. The first phase transition takes place and the SO(10) symmetry group breaks down to the $3_c 2_L 1_R 1_{B-L}$ symmetry group. As shown in Sec. 4.3, topologically stable monopoles form according to the Kibble mechanism [27] during this first phase transition.

Once the fields A, S, A' and S' have settled down to their minimum, since the first derivatives $\frac{\partial V}{\partial A}$, $\frac{\partial V}{\partial S}$, $\frac{\partial V}{\partial A'}$ and $\frac{\partial V}{\partial S'}$ are independent of Φ and $\mathcal{S}$, the fields A, A', S, and S' will stay in their minimum independently of what the fields Φ and $\mathcal{S}$ do. When the VEV of the inflaton field is greater than $\frac{\mu}{\sqrt{\alpha}} = s_c$, $|\Phi| = |\overline{\Phi}| = 0$, $F_{\mathcal{S}}$ term has a non-vanishing VEV, which means that supersymmetry is broken in the $\mathcal{S}$ direction, by an amount measured by the VEV of the $\mathcal{S}$ field. There is a non-vanishing vacuum energy density, $V = \mu^4$. An inflationary epoch (an exponentially extending universe) can start.

As has been pointed out recently [29], the fact that supersymmetry is broken for $|\mathcal{S}| > s_c$ implies that the one loop corrections to the effective potential are non-vanishing. They are given by [29]

$$\Delta V(\mathcal{S}) = \Sigma_i \frac{(-1)^F}{64\pi^2} M_i(\mathcal{S})^4 \ln\left(\frac{M_i(\mathcal{S})}{\Lambda^2}\right) \tag{4.21}$$

where the summation is over all helicity states for both fermions and bosons. Λ denotes a renormalisation mass and $(-1)^F$ indicates that the bosons and fermions make opposite sign contributions to the sum; (-1) stand for the fermions. Therefore the one loop effective potential obtained from equations (4.20) and (4.21) is given by [29],

$$\begin{aligned} V_{eff} &= \mu^4 [1 + \frac{\alpha^2}{32\pi^2} [2 \ln\left(\frac{\alpha^2 s^2}{\Lambda^2}\right) + \left(\frac{\alpha s^2}{\mu^2} - 1\right)^2 \ln\left(1 - \frac{\mu^2}{\alpha s^2}\right) \\ &+ \left(\frac{\alpha s^2}{\mu^2} + 1\right)^2 \ln\left(1 + \frac{\mu^2}{\alpha s^2}\right)] + \frac{m^2}{2\mu^4} s^2] \end{aligned} \tag{4.22}$$

where $s = |\mathcal{S}|$. Now $m \sim 10^3$ GeV and $\frac{\mu}{\sqrt{\alpha}} \sim 10^{15-16}$ GeV, hence unless $\alpha \ll 1$, the soft supersymmetry breaking term can be neglected. Its contribution to the scalar potential is negligible. For $s > s_c$, the quantum corrections to the effective potential help $\mathcal{S}$ to roll down its minimum. Below s_c, the $\mathcal{S}$ field is driven to zero by the positive term $\alpha^2|\mathcal{S}|^2|\Phi|^2$ which becomes larger with increasing $|\Phi|$. Rapidly the Φ, $\overline{\Phi}$ and $\mathcal{S}$ fields settle down the global minimum of the potential, corresponding to $< \Phi >_{\nu^c\nu^c} = < \overline{\Phi} >_{\overline{\nu^c\nu^c}} = \frac{\mu}{\sqrt{\alpha}}$ and $s = 0$. This does not affect the VEVs of the S, A, S', and A' fields which remain unchanged. The $3_c 2_L 1_R 1_{B-L}$ symmetry group breaks down to $3_c 2_L 1_Y Z_2$. As shown in Sec. 4.3, topological cosmic strings form during this phase transition. If inflation ends after the phase transition, the strings may be inflated away.

Inflation ends when the 'slow roll' condition is violated. The slow roll condition is characterised by [30]

$$\epsilon \ll 1 \; , \; \eta \ll 1, \tag{4.23}$$

where

$$\epsilon = \frac{M_{\rm pl}^2}{16\pi}(\frac{V'}{V})^2 \, , \qquad \eta = \frac{M_{\rm pl}^2}{8\pi}(\frac{V''}{V}) \tag{4.24}$$

and the prime refers to derivatives with respect to s. As pointed out by Copeland *et al.* [30], the slow-roll condition is a poor approximation. But as shown in [30], the number of e-foldings which occur between the time when η and ϵ reach unity and the actual end of inflation is a tiny fraction of unity. It is therefore sensible to identify the end of inflation with ϵ and η becoming of order unity.

From the effective potential (4.22) and the slow-roll parameters (4.24) we have [29]

$$\epsilon = \left(\frac{\alpha^2 M_{\rm pl}}{8\pi^2 M_{\rm G}}\right)^2 \frac{x^2}{16\pi}((x^2-1)\,\ln(1-\frac{1}{x^2}) + (x^2+1)\ln(1+\frac{1}{x}))^2 \tag{4.25}$$

$$\eta = \left(\frac{\alpha M_{\rm pl}}{2\pi M_{\rm G}}\right)^2 \frac{1}{16\pi}((3x^2-1)\,\ln(1-\frac{1}{x^2}) + (3x^2+1)\ln(1+\frac{1}{x}))^2 \tag{4.26}$$

where x is such that $\mathcal{S} = x\mathcal{S}_c$. The phase transition down to the standard model occurs when $x = 1$. The results are as follows. We find the values of the scalar coupling α, the scale $M_{\rm GUT}$ and the scale $M_{\rm G}$ which lead to successful inflation. For $\alpha \geq 35 - 43$, $M_{\rm G} \sim 10^{15} - 10^{16}$ GeV, ϵ is always greater than unity, and the slow roll condition is never satisfied. The scale $M_{\rm GUT}$ at which the monopoles form satisfies $M_{\rm pl} \geq M_{\rm GUT} \geq 10^{16} - 10^{17}$ GeV. For $\alpha \leq 0.02 - 0.002$ and $M_{\rm G} \sim 10^{15} - 10^{16}$ GeV, neither η nor ϵ ever reaches unity. $\mathcal{S}$ reaches $\mathcal{S}_c$ during inflation. Inflation must end by the instability of the Φ and $\overline{\Phi}$ fields. In that case, inflation ends in less than a Hubble time [30] once $\mathcal{S}$ reaches $\mathcal{S}_c$. Cosmic strings, which form when $x = 1$, are not inflated away. The scale $M_{\rm GUT}$ at which the monopoles form must satisfy $M_{\rm pl} \geq M_{\rm GUT} \geq 10^{16} - 10^{17}$ GeV. For the intermediates values of α, inflation occurs, and ends when either ϵ or η reaches unity; the string forming phase transition takes places once inflation completed.

4.6 Formation of cosmic strings

In this section we give general properties of the strings which form at the end of inflation when the $3_c 2_L 1_R 1_{B-L}$ symmetry group breaks down to $3_c 2_L 1_Y Z_2$. We find their width and their mass, give a general approach for their interactions with fermions and study their superconductivity.

4.6.1 General properties

Recall that, since the first homotopy group $\pi_1(\frac{3_c 2_L 1_R 1_{B-L}}{3_c 2_L 1_Y Z_2})$ is nontrivial, cosmic strings form during the second phase transition [see Eq. (4.7)] when the $3_c 2_L 1_R 1_{B-L}$ symmetry breaks down to $3_c 2_L 1_Y Z_2$. We note that the subspace spanned by R and $B-L$ is also spanned by X and Y. The generator of the string corresponds to the U(1) of SO(10) which commutes with SU(5), and the gauge field forming the string is the corresponding gauge field, which we call X. The strings are Abelian and physically viable. The model does not give rise to Alice strings, like most of the non-Abelian GUT phase transitions where Abelian and non-Abelian strings form at the same time. This is a good point of the model, since Alice strings give rise to quantum number non-conservation, and are therefore in conflict with the standard cosmology. The strings arising in our model can be related to the Abelian strings arising in the symmetry breaking pattern of SO(10) down to to the standard model with SU(5) $\times$ Z_2 as intermediate scale, since they have the same generator; the latter have been widely studied in the nonsupersymmetric case [55, 56] (see also Chap. 2). Nevertheless, in our model, inside the core of the string, we do not have an SO(10) symmetry restoration, but an $3_c 2_L 1_R 1_{B-L}$ symmetry restoration. We therefore don't have any gauge fields mediating baryon number violation inside the core of the strings, but one of the fields violates $B-L$. We also expect the supersymmetric strings to have different properties and different interaction with matter due to the supersymmetry restoration inside the core of the string. These special properties will be studied elsewhere.

The two main characteristics of the strings, their width and their mass, are determined through the Compton wavelength of the Higgs and gauge bosons forming the strings. The Compton wavelength of the Higgs and gauge bosons are respectively

$$\delta_{\Phi_{126}} \sim m^{-1}_{\Phi_{126}} = (2\,\alpha\, M_{\rm G})^{-1} \tag{4.27}$$

and

$$\delta_X \sim m_X^{-1} = (\sqrt{2}\, e\, M_{\rm G})^{-1}, \tag{4.28}$$

where e is the gauge coupling constant in supersymmetric SO(10) and it is given by $\frac{e^2}{4\pi} = \frac{1}{25}$ and $M_{\rm G}$ is the scale at which the strings form.

As mentioned above, the strings formed in our model can be related to those formed during the symmetry breaking pattern SO(10) $\to$ SU(5) $\times$ U(1) $\to$ SU(5) $\times$ Z_2. These strings have been studied by Aryal and Everett [55] in the nonsupersymmetric case. Using their results, with appropriate changes in the gauge coupling constant and in parameters of the Higgs potential, we find that the string mass per unit length of the string is given by

$$\mu \simeq (2.5 - 3) \times (M_{\mathrm{G}})^2, \tag{4.29}$$

for the scalar coupling α ranging from 5×10^{-2} to 2×10^{-1}. Recall that the mass per unit length characterises the entire properties of a network of cosmic strings.

4.6.2 No superconducting strings

One of the most interesting feature of GUT strings is their superconductivity. Indeed, if they become superconducting at the GUT scale, then vortons can form and dominate the energy density of the universe; the model loses all its interest. The strings arising in our model are not superconducting in Witten's sense [82]. They nevertheless can become current carrying with spontaneous current generation at the electroweak scale through Peter's mechanism [83]. But it is believed that this does not have any disastrous impact on the standard cosmology. It has been shown in the nonsupersymmetric case that the Abelian strings arising when SO(10) breaks down to SU(5) $\times$ Z_2 have right-handed neutrino zero modes [84]. Since the Higgs field forming the string is a Higgs boson in the **126** representation which gives mass to the right-handed neutrino and winds around the string, we expect the same zero modes on our strings. Since supersymmetry is restored in the core of the string, we also expect bosonic zero modes of the superpartner of the right-handed neutrino. Now, the question of whether or not the string will be current carrying will depend on the presence of a primordial magnetic field, and the quantum charges of the right-handed neutrino with respect to this magnetic field. If there is a primordial magnetic field under which the right-handed neutrino has a non-vanishing charge, then the current will be able to charge up. On the other hand, if such magnetic field does not exist, or if the right-handed neutrino is neutral, then there will be nothing to generate the current of the string. Although it is possible to produce a primordial magnetic field in a phase transition [85], we do not expect the fields produced through the mechanism of Ref. [85] to be able to charge up the current on the string, since the latter are correlated on too large scales. Nevertheless, the aim of this section is to show that the strings will not be superconducting at the GUT scale in any case. We can therefore assume a worse situation, that is, suppose that the magnetic fields are correlated on smaller scales, due to any mechanism for primordial magnetic field production any time after the Planck scale. In our model, cosmic strings form when $3_c 2_L 1_R 1_{B-L}$ breaks down to $3_c 2_L 1_Y Z_2$. Therefore the symmetric phase $3_c 2_L 1_Y Z_2$ will be associated with colour, weak and hypercharge magnetic fields. The colour and weak magnetic fields formed when SO(10) broke down to $3_c 2_L 1_R 1_{B-L}$, and and the hypercharge magnetic field formed at the following phase transition, formed from the R and $B-L$ magnetic fields. Since the charges of the right-handed neutrino with respect to the colour, weak and hypercharge magnetic fields are all vanishing, no current will be generated.

We conclude that the strings will not be superconducting at the GUT scale. They might become superconducting at the electroweak scale, but this does not seem to affect the standard big-bang cosmology in any essential way.

If the strings formed at the end of inflation are still present today, they would affect temperature fluctuations in the CBR and have affected large scale structure formation.

4.7 Observational consequences

We show here that the strings formed at the end of inflation may be present today. We find the scale $M_{\rm G}$ at which cosmic strings form and the scalar coupling of the inflaton field which are consistent with the temperature fluctuations observed by COBE. We then examine the dark-matter content of the model and make a qualitative discussion regarding large scale structure formation.

4.7.1 Temperature fluctuations in the CBR

If both inflation and cosmic strings are part of the scenario, temperature fluctuations in the CBR are the result of the quadratic sum of the temperature fluctuations from inflationary perturbations and cosmic strings.

The scalar density perturbations produced by the inflationary epoch induce temperature fluctuations in the CBR which are given by [29]

$$\left(\frac{\delta T}{T}\right)_{inf} \simeq \sqrt{\frac{32\pi}{45}}\frac{V^{\frac{3}{2}}}{V' M_{\rm pl}^3}|_{x_q} \tag{4.30}$$

$$\approx (8\pi N_q)^{\frac{1}{2}}\left(\frac{M_{\rm G}}{M_{\rm pl}}\right)^2, \tag{4.31}$$

where the subscript indicates the value of $\mathcal{S}$ as the scale (which evolved to the present horizon size) crossed outside the Hubble horizon during inflation, and N_q ($\sim 50-60$) denotes the appropriate number of e-foldings. The contribution to the CBR anisotropy due to gravitational waves produced by inflation in this model is negligible.

The cosmic strings density perturbations also induce CBR anisotropies given by [81]

$$\left(\frac{\delta T}{T}\right)_{c.s.} \approx 9\, G\mu, \tag{4.32}$$

where μ is the strings mass per unit length, which is given by Eq. (4.29). It depends on the scalar coupling α. Since the later is undetermined, we can use the order of magnitude

$$\mu \sim \eta^2, \tag{4.33}$$

which holds for a wide range of the parameter α; see Eq. (4.29) and in Ref. [55]. In Eq. (4.33), η is the symmetry breaking scale associated with the strings formation, here $\eta = M_{\rm G}$.

Hence, from equations (4.31) and (4.32) the temperature fluctuations in the CBR are given by

$$\left(\frac{\delta T}{T}\right)_{tot} \approx \sqrt{\left(\frac{\delta T}{T}\right)_{inf}^2 + \left(\frac{\delta T}{T}\right)_{c.s.}^2} \tag{4.34}$$

$$\approx \sqrt{8\pi N_q + 81} \left(\frac{M_{\rm G}}{M_{\rm pl}}\right)^2 . \tag{4.35}$$

The temperature fluctuations from both inflation and cosmic strings add quadratically. Since they are both proportional to $\frac{M_{\rm G}}{M_{\rm pl}}$ their computation is quite easy.

An estimate of the coupling α is obtained from the relation [29]

$$\frac{\alpha}{x_q} \sim \frac{8\pi^{\frac{3}{2}}}{\sqrt{N_q}} \frac{M_{\rm G}}{M_{\rm pl}}. \tag{4.36}$$

With $x_q \sim 10$, using Eqs. (4.35) and (4.36) and using the temperature fluctuations measured by COBE $\simeq 1.3 \times 10^{(-5)}$ [16] we get

$$\alpha \simeq 0.03, \tag{4.37}$$
$$M_{\rm G} \simeq 6.7 \times 10^{15}\ \mathrm{GeV}\ . \tag{4.38}$$

With these values, we find that η reaches unity when $x \simeq 1.4$ and the scale $M_{\rm GUT}$ at which the monopoles form must satisfy,

$$M_{\rm pl} \geq M_{\rm GUT} \geq 6.7 \times 10^{16}\ \mathrm{GeV} \tag{4.39}$$

where $M_{\rm pl}$ is the Planck mass $\simeq 1.22 \times 10^{19}$ GeV.

From the above results, we can be confident that the strings forming at the end of inflation should still be around today.

Now that we have got values for the scalar coupling α and the scale $M_{\rm G}$ at which the strings form, the Compton wavelength of the Higgs and gauge bosons forming the strings given by Eqs. (4.27) and (4.28) can be computed. We find

$$\delta_{\Phi_{126}} \sim m_{\Phi_{126}}^{-1} \sim 0.42 \times 10^{-28}\ \mathrm{cm} \tag{4.40}$$

for the Compton wavelength of the Higgs field forming the string and

$$\delta_X \sim m_X^{-1} \sim 0.29 \times 10^{-29}\ \mathrm{cm} \tag{4.41}$$

for the Compton wavelength of the strings gauge boson. $\delta_{\Phi_{126}} > \delta_X$ thus the strings possess an inner core of false vacuum of radius $\delta_{\Phi_{126}}$ and a magnetic flux tube with a smaller radius δ_X. The string energy per unit length is given by Eq. (4.29), thus, using above results, we have

$$G\mu \sim 7.7 \times 10^{-7}, \tag{4.42}$$

where G is Newton's constant. The results are slightly affected by the number of e-folding and by the order of magnitude (4.33) used to compute the temperature fluctuations in the CBR due to cosmic strings in Eqs. (4.34) and (4.35). Once we have found the value for the scalar coupling α for successful inflation, we can redo the calculations with a better initial value for the string mass per unit length; see Eq. (4.29); the scalar coupling α is unchanged. The results are summarised in Table 4.1.

Nq	50	50	60	60
μ_{init}	η^2	$2.5\,\eta^2$	η^2	$2.5\,\eta^2$
$M_{\rm G}$	6.7×10^{15}	6.3×10^{15}	6.5×10^{15}	6.1×10^{15}
α	0.03	0.03	0.03	0.29
δ_Φ	0.42×10^{-28}	0.44×10^{-28}	0.43×10^{-28}	0.46×10^{-28}
δ_X	0.29×10^{-29}	0.31×10^{-29}	0.30×10^{-29}	0.32×10^{-29}
$G\mu$	7.7×10^{-7}	6.7×10^{-7}	7.1×10^{-7}	6.3×10^{-7}

Table 4.1 : The table shows the values obtained for the scale $M_{\rm G}$ at which the strings form, the scalar coupling α, the Higgs and gauge boson Compton wavelengths δ_Φ and δ_X of the strings, and $G\mu$, where μ is the strings mass per unit length and G is the Newton's constant, for different values of the number of e-foldings N_q and for different initial values used for their computation for the string mass-per-unit length.

4.7.2 Dark matter

We specify here the nature of dark matter generated by the model.

If we go back to the symmetry breaking pattern of the model given by Eq. (4.7), we see that a discrete Z_2 symmetry remains unbroken down to low energy. This Z_2 symmetry is a subgroup of both the Z_4 centre of SO(10) and of $\mathrm{U}(1)_{B-L}$ subgroup of SO(10). This Z_2 symmetry acts as matter parity. It preserves large values for the proton lifetime and stabilises the lightest superparticle. The LSP is a good cold dark matter candidate.

The second stage of symmetry breaking in Eq. (4.7) is implemented with the use of a $\mathbf{126}+\overline{\mathbf{126}}$ pair of Higgs multiplets, with VEVs in the direction of the $\mathrm{U}(1)_X$ of SO(10) which commutes with SU(5). The $\overline{\mathbf{126}}$ multiplet can couple with fermions and give superheavy Majorana mass to the right-handed neutrino, solving the solar neutrino problem via the MSW mechanism [40] and providing a good hot dark matter candidate. This can be done if all fermions are assigned to the 16-dimensional spinorial representation of SO(10). In that case, couplings of the form $f\overline{\Psi}\Psi\overline{\mathbf{126}}$, where Ψ denotes a 16-dimensional spinor to which all fermions belonging to a single family are assigned, provide right-handed neutrinos masses of order $m_R \simeq 10^{12}$ GeV, if $f\sim 10^{-4}$ GeV. Neutrinos also get Dirac masses which are typically of the order of the mass of the up-type quark of the corresponding family; for instance $m_D^{\nu_e}\simeq m_u$. After diagonalising the neutrino mass matrix, one finds that the right-handed neutrino mass $m_{\nu_R}\simeq m_R$ and the left-handed neutrino mass $m_{\nu_L}\simeq \frac{m_D^2}{m_R}$. With the above values we get

$$m_{\nu_e} \sim 10^{-7}\,\mathrm{eV} \tag{4.43}$$
$$m_{\nu_\mu} \sim 10^{-3}\,\mathrm{eV} \tag{4.44}$$
$$m_{\nu_\tau} \sim 10\,\mathrm{eV} \tag{4.45}$$

The tau neutrino is a good hot dark matter candidate.

Our model thus provides both CDM and HDM and is consistent with mixed cold and hot DM scenarios.

It is interesting to note that CDM and HDM are, in this model, related to each other. Indeed, the Z_2 symmetry in Eq. (4.7), which stabilises the LSP, is kept unbroken because a $\mathbf{126} + \overline{\mathbf{126}}$ and not $\mathbf{16} + \overline{\mathbf{16}}$ pair of Higgs fields are used to break $\mathrm{U}(1)_{B-L}$. If a 16 dimensional Higgs representation were used, the right-handed neutrino could not get a superheavy Majorana mass and thus no HDM could be provided, also the Z_2 symmetry would have been broken, and thus the LSP destabilised. The $\mathbf{126} + \overline{\mathbf{126}}$ pair of Higgs fields provide superheavy Majorana to the right-handed neutrino and keeps the Z_2-parity unbroken. It leads to both HDM and CDM. We conclude that, in this model, CDM and HDM are intimately related. Either the model provides both cold and hot dark matter, or it does not provide any. Our model provides both CDM and HDM.

4.7.3 Large scale structure

We give here only a qualitative discussion of the consistency of the model with large scale structure. We do not make any calculations which would require a full study on their own. We can nevertheless use various results on large scale structure with inflation or cosmic strings. Since we determined the nature of dark matter provided by the model, we may make sensible estimations about the the consistency of the model with large scale structure.

Presently there are two candidates for large scale structure formation, the inflationary scenario and the topological defects scenario with cosmic strings. Both scenarios are always considered separately. Indeed, due to the difference in the nature of the density perturbations in each of the models, density perturbation calculations due to a mixed strings and inflation scenario are not straightforward. Indeed in the inflation-based models density perturbations are Gaussian adiabatic whereas in models based on topological defects inhomogeneities are created in an initially homogeneous background [86].

In the attempt to explain large scale structure, inflation-seeded cold dark matter models or strings models with HDM are the most capable [86]. In adiabatic perturbations with hot dark matter small scale perturbations are erased by free streaming whereas seeds like cosmic strings survive free streaming. Small scale fluctuations in models with cosmic strings and HDM are not erased, but their growth is only delayed by free streaming [87].

Our model involves both hot and cold dark matter, and both inflation and cosmic strings. It is therefore sensible to suggest that our model will be consistent with large scale structure formation, with the large scale structures resulting from the inflationary scenario and small scale structures (galaxies and clusters) being due to cosmic strings.

4.8 Conclusions

We have successfully implemented a false vacuum hybrid inflationary scenario in a supersymmetric SO(10) model. We first argued that this type of inflationary scenario is a natural way for inflation to occur in global supersymmetric SO(10) models. It is natural, in the sense that the inflaton field emerges naturally from the theory, no external field and no external symmetry has to be added. The scenario does not require any fine tuning. In our specific model, the SO(10) symmetry is broken via the intermediate $3_c 2_L 1_R 1_{B-L}$ symmetry down to the standard model with unbroken matter parity $3_c 2_L 1_Y Z_2$. The model gives a solution for the doublet-triplet splitting via the Dimopoulos-Wilczek mechanism. It also suppresses rapid proton decay .

The inflaton, a scalar field singlet under SO(10), couples to the Higgs mediating the phase transition associated with the breaking of $3_c 2_l 1_R 1_{B-L}$ down to the standard model. The scenario starts with chaotic initial conditions. The SO(10) symmetry breaks at M_{GUT} down to $3_c 2_L 1_R 1_{B-L}$ and topologically stable monopoles form. There is a non-vanishing vacuum energy density, supersymmetry is broken, and an exponentially extending epoch starts. Supersymmetry is broken, and therefore quantum corrections to the scalar potential can not be neglected. The latter help the inflaton field to roll down its minimum. At the end of inflation the $3_c 2_L 1_R 1_{B-L}$ breaks down to $3_c 2_L 1_Y Z_2$, at a scale M_{G}, and cosmic strings form. They are not superconducting.

Comparing the CBR temperature anisotropies measured by COBE with that predicted by the mixed inflation-cosmic strings scenario, we find values for the scalar coupling α and for the scale M_{G} at which the strings form. M_{GUT} is calculated such that we get enough e-foldings to push the monopoles beyond the horizon. The results are summarised in Table 4.1. The evolution of the strings is that of topologically stable cosmic strings. The model is consistent with a mixed HCDM scenario. Left-handed neutrinos get very small masses and the tau neutrino may serve as a good HDM candidate. They could also explain the solar neutrino problem via the MSW mechanism. The unbroken matter parity stabilises the LSP, thus providing a good CDM candidate. A qualitative discussion leads to the conclusion that the model is consistent with large scale structures, very large scale structures being explained by inflation and cosmic strings explaining structures on smaller scales. An algebraic investigation for this purpose would be useful, but will require further research.

Chapter 5

New Mechanism for Leptogenesis

5.1 Introduction

Big-bang nucleosynthesis predicts the present abundances of the light-elements He^3, D, He^4 and Li^7 as a function of an adjustable parameter, the baryon-to-photon ratio $\eta = \frac{n_B}{n_\gamma}$. Recent analysis [11] shows that η must lie in the range

$$\eta \simeq (2-7) \times 10^{-10} \tag{5.1}$$

to predict the light elements abundances which agree with observations. η is related to the baryon number of the universe $B = \frac{n_B}{s}$ via $\frac{n_B}{n_\gamma} = 1.80 g_* B$, where s is the entropy of the universe and g_* counts the number of massless degrees of freedom. According to the big-bang cosmology, the universe started on a baryon-symmetric state. Hence there must have been out-of-equilibrium processes in the very early universe which violated baryon number plus C and CP [88] to explain the matter/antimatter asymmetry of the universe today.

The standard scenario for baryogenesis is provided by grand unified theories (GUTs), via the out-of-equilibrium decays of heavy gauge and Higgs bosons which violate baryon number (B) and/or lepton number (L). But it has been realized a decade ago that, due to an anomaly in the baryonic current, and due to the non trivial structure of the vacuum in non Abelian gauge theories, any baryon asymmetry generated at the grand unified scale would be erased by the electroweak anomaly [89]. Baryon and lepton number non-conservation in the standard electroweak theory had been known for a while [90]. But in 1976, 't Hooft [90] pointed out that the quantum tunnelling transition rate between two topologically different vacua by instantons is exponentially suppressed by the WKB factor $\exp(\frac{-4\pi}{\alpha_w})$ at zero temperature. It is only in 1985, after the discovery of sphalerons [91], static but unstable solutions of the classical field equations in the electroweak theory, that Kuzmin, Rubakov and Shaposhnikov [89] realized that at high temperature, the transition rate between two neighbouring inequivalent vacua is unsuppressed. Since vacuum to vacuum transitions in the electroweak theory conserve $B-L$ but violate $B+L$

and since B and L are related to $B+L$ and $B-L$ by

$$B = \frac{1}{2}(B+L) + \frac{1}{2}(B-L) \tag{5.2}$$

$$L = \frac{1}{2}(B+L) - \frac{1}{2}(B-L)\,, \tag{5.3}$$

and therefore B and L are proportional to $B-L$:

$$< B >_T = \alpha < (B-L) >_T \tag{5.4}$$

$$< L >_T = \gamma < (B-L) >_T, \tag{5.5}$$

with α and $-\gamma \simeq 0.5$. Weak interactions only involve left-handed fermion fields and hence the factors α and $-\gamma$ are not exactly $\frac{1}{2}$ [92]. We see from Eqs. (5.4) and (5.5) that, unless the universe started with a non-vanishing $B-L$ asymmetry, any B or L asymmetry generated at the grand unified scale will be erased by the electroweak anomaly. GUTs such as SU(5) conserve $B-L$, and hence fail in explaining the baryon number of the universe today.

An initial $B-L$ asymmetry can be obtained in theories containing an extra gauge $\mathrm{U}(1)_{B-L}$ symmetry, such as the simple U(1) extension of the standard model $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)_{Y'}$, where the charge Y' is a linear combination of Y and $B-L$, left-right models or SO(10) and E(6) GUT's. These theories predict one or more extra fermions in addition to the usual quarks and leptons. One of them, singlet under the standard model gauge group, can be interpreted as a right-handed neutrino. Right-handed neutrinos acquire a heavy Majorana mass at the scale of $B-L$ breaking. Out-of-equilibrium decays of heavy Majorana right-handed neutrinos may provide the necessary primordial $B-L$ asymmetry, and hence explain the observed baryon asymmetry today [23, 24, 25]. This mechanism however requires either very heavy neutrinos or extreme fine tuning of the parameters in the neutrino mass matrix [25]. Also, the masses of the new gauge bosons must be bigger than the smallest heavy neutrino mass [93]. Hence there is a wide range of parameters for which the mechanism does not produce enough baryon asymmetry.

In this chapter, we show that in unified models involving an extra gauge $\mathrm{U}(1)_{B-L}$ symmetry, a primordial $B-L$ asymmetry can be generated by the out-of-equilibrium decays of right-handed neutrinos released by collapsing cosmic string loops. As a consequence of $\mathrm{U}(1)_{B-L}$ breaking, cosmic strings may form at the $B-L$ breaking scale according to the Kibble mechanism [27]. We call them $B-L$ cosmic strings. The Higgs field mediating the breaking of $B-L$ is the Higgs field forming the strings and it is the same Higgs field that gives a heavy Majorana mass to the right-handed neutrinos. Hence, due to the winding of the Higgs field around the string, we expect right-handed neutrino zero modes [94] trapped in the core of the strings. These zero modes are predicted by an index theorem [95]. There are also modes of higher energy bounded to the strings. We shall consider only the zero modes, which are the most favourable to be trapped. $B-L$ cosmic string loops lose their energy by emitting gravitational radiation and rapidly shrink to a point, releasing these right-handed neutrinos. This is an out-of-equilibrium process. Right-handed neutrinos acquire a heavy Majorana mass and decay into massless leptons and electroweak Higgs particles to produce a lepton asymmetry. This lepton asymmetry is converted into a baryon asymmetry via sphaleron transitions.

5.2 Unified theories with $B-L$ cosmic strings

Theories beyond the standard model gauge group with rank greater or equal to five and containing an extra $\mathrm{U}(1)_{B-L}$ gauge symmetry predict fermions in addition to the usual quarks and leptons of the standard model. One of them, singlet under the standard model gauge group, may be interpreted as a right-handed neutrino. Right-handed neutrinos may acquire a heavy Majorana mass at the $B-L$ breaking scale. The simplest such extension of the standard model is the $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)_{Y'}$ symmetry group where the charge Y' is a linear combination of Y and $B-L$. Left-right models and grand unified theories with rank greater or equal to five also contain a $\mathrm{U}(1)_{B-L}$ gauge symmetry and predict right-handed neutrinos. In such theories, as a consequence of $\mathrm{U}(1)_{B-L}$ breaking, cosmic strings may form. We call them $B-L$ cosmic strings.

Topological $B-L$ cosmic strings form when a gauge group $\mathrm{G} \supset \mathrm{U}(1)_{B-L}$ breaks down to a subgroup $\mathrm{H} \not\supset \mathrm{U}(1)_{B-L}$ of G, if the vacuum manifold $\frac{\mathrm{G}}{\mathrm{H}}$ is simply connected, that is if the first homotopy group $\pi_1(\frac{\mathrm{G}}{\mathrm{H}})$ is non-trivial. If $\pi_1(\frac{\mathrm{G}}{\mathrm{H}}) = I$ but string solutions still exist, then embedded strings [78, 96] form when G breaks down to H. Embedded strings are stable for a wide range of parameters. In left-right models, embedded $B-L$ strings usually form. In the simple U(1) extension of the standard model $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)_{Y'}$ where Y' is a linear combination of Y and $B-L$ topological strings form. In grand unified theories with rank greater than five, such as SO(10) or E(6), $B-L$ cosmic strings may form, depending on the symmetry breaking pattern and on the set of Higgs fields used to do the breaking down to the standard model gauge group [54, 82]. There is a wide range of theories which contain both $\mathrm{U}(1)_{B-L}$ and $B-L$ cosmic strings.

5.3 Right-handed neutrino zero modes in $B-L$ cosmic strings

We noticed that in unified theories with rank greater or equal to five which contain an extra $\mathrm{U}(1)_{B-L}$ gauge symmetry $B-L$ cosmic strings often form. Consider then a unified model with such a $\mathrm{U}(1)_{B-L}$ symmetry and stable $B-L$ cosmic strings. Such a theory predicts right-handed neutrinos.

The gauge and Higgs fields forming the strings will be the $B-L$ associated gauge boson A' and the Higgs field ϕ_{B-L} used to break $\mathrm{U}(1)_{B-L}$. Right-handed neutrinos acquire heavy Majorana mass via Yukawa couplings to ϕ_{B-L}. So ϕ_{B-L} both winds around the strings and gives heavy Majorana mass to the right-handed neutrinos. Thus, following Jackiw and Rossi[94], we expect right-handed neutrino zero modes in the core of $B-L$ cosmic strings. The $\mathrm{U}(1)_{B-L}$

part of the theory is described by the Lagrangian

$$\begin{aligned} L &= \frac{1}{4} f_{\mu\nu} f^{\mu\nu} + (D_\mu \phi_{B-L})^\dagger (D^\mu \phi_{B-L}) - V(\phi_{B-L}) \\ &+ i\overline{\nu}_L \gamma^\mu D_\mu \nu_L + i\overline{\nu}_R \gamma^\mu D_\mu \nu_R \\ &+ i\lambda \phi_{B-L} \overline{\nu}_R \nu_L^c - i\lambda \phi^*_{B-L} \overline{\nu}^c_L \nu_R + L_f \,. \end{aligned} \tag{5.6}$$

The covariant derivative $D_\mu = \partial_\mu - iea^{Y'}_\mu$ where e is the gauge coupling constant and Y' is a linear combination of $B-L$ and Y. λ is a Yukawa coupling constant, and $V(\phi_{B-L})$ is the Higgs potential. The spinor $N = \nu_R + \nu_L^c$ is a Majorana spinor satisfying the Majorana condition $N^c \equiv C\gamma_0^T N^* = N$, where C is the charge conjugation matrix. Hence N has only two independent components, two degrees of freedom. L_f is the fermionic $B-L$ part of the Lagrangian which does not contain neutrino fields.

For a straight infinite cosmic string lying along the z-axis, the Higgs field ϕ_{B-L} and the Y' gauge field a in polar coordinates (r, θ) have the form

$$\phi_{B-L} = f(r) e^{in\theta} \tag{5.7}$$

$$a_\theta = -n\tau \frac{g(r)}{er} \qquad a_z = a_r = 0, \tag{5.8}$$

where n is the winding number; it must be an integer. Most strings have winding number $n = 1$; strings with winding number $|n| > 1$ are unstable. τ is the string's generator; it is the normalised $\mathrm{U}(1)_{Y'}$ generator. It has different eigenvalues for different fermion fields. The functions $f(r)$ and $g(r)$ must satisfy the following boundary conditions

$$f(0) = 0 \qquad \text{and} \qquad f \to \eta_{B-L} \quad \text{as} \quad r \to \infty, \tag{5.9}$$

$$g(0) = 0 \qquad \text{and} \qquad g \to 1 \quad \text{as} \quad r \to \infty\,, \tag{5.10}$$

where η_{B-L} is the scale of $B-L$ breaking. The exact forms of the functions $f(r)$ and $g(r)$ depend on the Higgs potential $V(\phi_{B-L})$.

From the Lagrangian (5.6) we derive the equation for the right-handed neutrino field:

$$i\gamma^\mu D_\mu \nu_L^c - i\lambda \phi^*_{B-L} \overline{\nu}^c_L = 0 \tag{5.11}$$

where $\nu_L^c = C\gamma_0^T \nu_R^*$. Solving (5.11), we find that Majorana neutrinos trapped as tranverse zero modes in the core of $B-L$ cosmic strings have only one independent component. For an $n = 1$ vortex it takes the form :

$$N_1 = \beta(r, \theta)\, \alpha(z + t) \tag{5.12}$$

where $\beta(r, \theta)$ is a function peaked at $r = 0$ which exponentially vanishes outside the core of the string, so that the fermions effectively live on the strings. The z and t dependence of α shows that the neutrinos travel at the speed of light in the $-z$ direction, so that they are effectively massless. In a $n = -1$ vortex, the function $\alpha = \alpha(t - z)$, so that the fermions travel at the speed of light in the $+z$ direction. These fermions can be described by an effective theory in $1 + 1$ dimensions. The usual energy to momentum relation

$$E = P \tag{5.13}$$

holds. We have no boundary conditions in the 1 spatial dimension, and the spectrum of states is continuous. In the ground state the Fermi momentum of the zero modes is $p_F = 0$.

The field solution (5.12) and the energy to momentum relation (5.13) have been derived for fermions on a straight infinite string. However physical cosmic strings are very wiggly and are not straight. Hence relations (5.12) and (5.13) do not hold in the physical case. Neither do they hold for cosmic string loops, even if the latter are assumed to be smooth. The behaviour of Dirac fermions on a cosmic string of finite radius of curvature has been analysed by Barr and Matheson[97]. On such strings, fermions are characterised by their angular momentum L. The energy relation becomes[97]

$$E = \frac{(L + \frac{1}{2})}{R} = P + \frac{1}{2R} \tag{5.14}$$

and hence the energy spectrum is

$$E = \frac{\pm(\mathrm{n} + \frac{1}{2})}{R} \tag{5.15}$$

where $n \in \mathbb{N}$. We see from Eqs. (5.13), (5.14), and (5.15) that, when R is very large, the string looks locally like a straight string. We have an almost continuous spectrum of states. The fact that the string gets a finite curvature acts as a perturbation on the string bound states. The energy levels get quantised and the Fermi energy gets a non-vanishing value $E_F = \frac{1}{2R}$. As the string loop shrinks, its radius R decreases and we see from Eq. (5.15) that the Fermi energy increases and that the separation between energy levels gets wider.

5.4 Leptogenesis via decaying cosmic string loops

Cosmic string loops form via the intercommuting of long strings. Some loops are formed when the network initially forms. Cosmic strings loops lose their energy via gravitational radiation and rapidly decay, releasing right-handed neutrinos trapped as transverse zero modes in their core.

Assuming that a loop decays when its radius R becomes comparable to its width $w \sim \eta_{B-L}^{-1}$, we deduce that the Fermi energy level when the loop decays is $E_F \sim \frac{1}{2}\eta_{B-L}$, where the $B - L$ breaking scale η_{B-L} is proportional to the right-handed neutrino mass. E_F is lower than the energy needed by a neutrino to escape the string [97]. Hence, when a cosmic string loop decays, it releases at least $n_\nu = 1$ heavy Majorana neutrinos. Quantum fluctuations and finite temperature corrections may increase n_ν. Part of the final burst of energy released by the decaying cosmic string loop is converted into mass energy for the gauge and Higgs particles released by the string, and into mass energy for the neutrinos. A decaying $B - L$ cosmic string loop releases heavy $B - L$ Higgs particles which can decay into right-handed neutrino pairs, and hence increase n_ν. This is an out-of-equilibrium process. Due to angular momentum conservation, the massive Majorana neutrinos released by a decaying cosmic string loop which were trapped as transverse zero modes are spinning particles.

Heavy Majorana right-handed neutrinos interact with the standard model leptons via the Yukawa couplings

$$L_Y = h_{ij}\overline{l_i}H_{ew}\nu_{Rj} + h.c. \tag{5.16}$$

where l is the usual lepton doublet; for the first family $l = (e,\nu)_L$. H_{ew} is the standard model doublet of Higgs fields. Majorana right-handed neutrinos can decay via the diagrams shown in Figs.1.a. and 1.b.

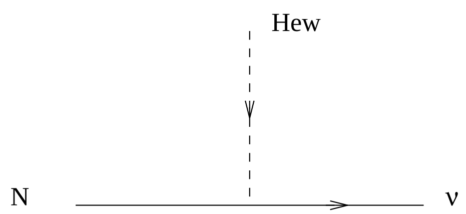

FIG. 1a. The tree-level diagram for right-handed neutrino decay

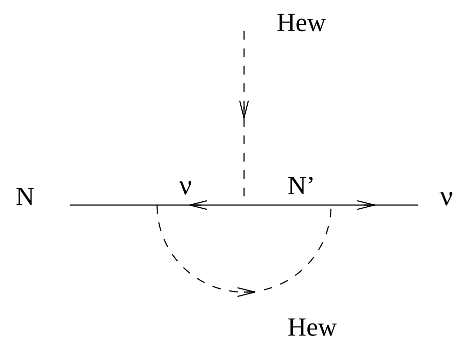

FIG. 1b. The one loop diagram for right-handed neutrino decay

CP is violated through the one loop radiative correction involving a Higgs particle as shown in figure 1.b., that is, there is a difference between the branching ratio of N to the $\nu_L H_{ew}^{0*}$ final state and the branching ratio of N to the $\overline{\nu_L}H_{ew}^0$ final state. The right-handed neutrinos are out-of-equilibrium, and hence a lepton asymmetry can be generated. The lepton asymmetry is characterised by the CP violation parameter ϵ which, assuming that the neutrino Dirac masses fall into a hierarchical pattern qualitatively similar to that of the leptons and quarks, is estimated to be [24]

$$\epsilon \simeq \frac{m_{D_3}^2}{\pi v^2}\frac{M_{N1}}{M_{N2}}\sin\delta \tag{5.17}$$

where m_{D_3} is the Dirac mass of the third lepton generation, v is the vacuum expectation value of the electroweak Higgs field $v =< H_{ew} >= 174$ GeV, M_{N1} and M_{N2} are the right-handed neutrino Majorana masses of the first and second generation respectively and δ is the CP violating phase.

The corresponding $B-L$ asymmetry (we use $B-L$ instead of L since the $(B+L)$-violating electroweak anomaly conserves $B-L$) must be calculated solving Boltzmann equations which take into account all B, L and $B+L$ violating interactions and their inverse decay rates. We can however calculate the $B-L$ asymmetry produced taking into account only the out-of-equilibrium decays of right-handed neutrinos released by decaying cosmic string loops and assuming that the rates of inverse decays are negligible. Hence an upper limit on the baryon number per commoving volume at temperature T is then given by [5]

$$B(T) = \frac{1}{2}\frac{N_\nu(t)\epsilon}{s}, \tag{5.18}$$

where s is the entropy at time t and $N_\nu(t)$ is the number density of right-handed neutrinos which have been released by decaying cosmic string loops at time t. Recall that the temperature T is related to the cosmic time t via the relation

$$t = 0.3\, g_*^{-\frac{1}{2}}\frac{M_{\rm pl}}{T^2}, \tag{5.19}$$

where g_* counts the number of massless degrees of freedom in the corresponding phase and $M_{\rm pl}$ is the Planck mass. s, the entropy at time t, is given by

$$s = \frac{2}{45}\pi^2 g_* T^3. \tag{5.20}$$

$N_\nu(t)$ is approximately n_ν times the number density of cosmic string loops which have shrinked to a point at temperature T. Assuming that sphaleron transitions are not in thermal equilibrium below T_{ew}, and neglecting any baryon number violating processes which might have occured below T_{ew}, the baryon number of the universe at temperature $T \leq T_{ew}$ is then given by

$$B = B(T_{ew}), \tag{5.21}$$

which is also the baryon number of the universe today. If sphaleron transitions are also rapid below T_{ew}, we should include the neutrinos released below T_{ew}. However, below T_{ew} the number density of decaying cosmic string loops is negligible, and hence this would not affect the result in any sense.

The number density of decaying cosmic string loops can be estimated from the three scales model of Ref. [98]. The model is based on the assumption that the cosmic string network evolution is characterised by three length scales $\xi(t)$, $\overline{\xi}(t)$ and $\chi(t)$ related to the long string density, the persistence length along the long strings (which is related to the fact that the typical loop size is much smaller than ξ), and the small scale structure along the strings respectively.

Cosmic string loops lose their energy by emitting gravitational radiation at a rate [98] $\dot{E} = -\Gamma_{loops} G\mu^2$ where $\mu \sim T_c^2$ is the string mass-per-unit-length and $T_c = \eta_{B-L}$ is the critical temperature of the phase transition leading to the string network formation. G is Newton's constant. The numerical factor $\Gamma_{loops} \sim 50-100$ depends on the loop's shape and trajectory, but is independent of its length. The mean size of a loop born at t_b is assumed to be $(k-1)\Gamma_{loops} G\mu t_b$. At a later time t, it is then $\Gamma_{loops} G\mu(kt_b - t)$. The loop finally disappears at a time $t = kt_b$. Numerically, k is found to lie between 2 and 10.

The rate at which the string loops form in a volume V is given by [98]

$$\dot{N}(t_b) = \frac{\nu V}{(k-1)\Gamma_{loops} G\mu t_b^4} \tag{5.22}$$

where the parameter ν can be expressed in terms of the various length scales of the model which vary with time. We start with $\xi \sim \overline{\xi} \sim \chi$. Then ξ and later $\overline{\xi}$ will start to grow and will evolve to the scaling regime characterised by $\xi(t)$ and $\overline{\xi}(t) \sim t$. The length scale χ grows much less rapidly. Therefore ν varies with time. In the scaling regime, ν is estimated to lie in the range $\nu = 0.1 - 10$ [98].

Note that it has recently been shown that cosmic string networks reach the scaling solution at a time t_* much smaller than previously estimated [99]. The authors of ref.[99] find that in the radiation dominated era

$$t_* = \beta^2 f^3 \frac{M_{\rm pl}^{-1}}{(G\mu)^2} \tag{5.23}$$

where β is a numerical factor related to the number of particle species interacting with the strings (expected to be of order unity for minimal GUT strings) and $f = 0.3\, g_*^{-\frac{1}{2}}$ where g_* counts the number of massless degrees of freedom after the H phase (which will usually be the $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ phase). For minimal GUT strings $t_* = 8 \times 10^2\, t_c$ [99], t_c is the time at which the strings form, and the associated temperature is $T_* \simeq 10^{14.5}$ GeV. Hence our approximation for the rate of cosmic string loop formation is suitable; it leads to a lower bound on the number of cosmic string loops. Only numerical simulations could lead us to a better estimate, but it is beyond the scope of this analysis.

Since it has been shown that most of the loops formed have relatively small size, we shall assume that the number of loops rejoining the network is negligible and thus that the number of decaying loops is equal to the number of forming loops. Hence the number density of right-handed neutrinos which have been released by decaying cosmic string loops at time t is given by

$$N_\nu(t) = n_\nu \int_{kt_c}^{t} \frac{\dot{N}(t_b)}{V} \Big(\frac{r(t_b)}{r(t)}\Big)^3 dt_b \tag{5.24}$$

where where $r(t)$ is the cosmic scale factor and n_ν is the mean number of right-handed neutrinos released by a single decaying loop. In the radiation dominated era $r(t) \sim t^{\frac{1}{2}}$. After integration we obtain

$$N_\nu(t) = \frac{2}{3} n_\nu \,\frac{\nu}{(k-1)\Gamma_{loops} G\mu}\, \frac{1}{(0.3)^3 g_*^{-\frac{3}{2}}}\, \Big[\frac{1}{k^{\frac{3}{2}}}(\frac{T_c}{M_{\rm pl}})^3 - (\frac{T}{M_{\rm pl}})^3\Big] T^3 \tag{5.25}$$

where we have used the fact that the cosmic time t is related to the universe temperature via Eq. (5.19). Hence the baryon number per commoving volume today produced by the decays of heavy Majorana right-handed neutrinos released by decaying cosmic string loops given in Eq. (5.18) becomes

$$B \simeq \frac{7.5 g_*^{\frac{1}{2}}}{(0.3\,\pi)^3}\, \frac{\nu}{(k-1)k^{\frac{3}{2}}\Gamma_{loops}} \frac{T_c}{M_{\rm pl}}\, \frac{m_{D_3}^2}{v^2}\, \frac{M_{N1}}{M_{N2}} \,\sin\delta, \tag{5.26}$$

where we have used Eq. (5.17) for the CP violation parameter ϵ. The produced B asymmetry depends on the cosmic string scenario parameters, on the neutrino mass matrix and on the strength of CP violation.

We now calculate the lower and upper bounds on B which correspond to different values of the parameters in the cosmic string scenario. We fix the neutrino mass matrix parameters and assume maximum CP violation, i.e. $\sin\delta = 1$. We assume that the Dirac mass of the third generation fermions lies in the range $m_{D_3} = 1 - 100$ GeV, and that the ratio of the right-handed neutrino masses of the first and second generation $\frac{M_{N1}}{M_{N2}} = 0.1$. With the above assumptions, we find B to lie in the range

$$B \simeq (1 \times 10^{-10} - 5 \times 10^{-2})\, g_*^{\frac{1}{2}}\, (\frac{T_c}{M_{\rm pl}}), \tag{5.27}$$

and we see that B strongly depends on the cosmic string scenario parameters. Recall that the baryon number-to-photon ratio $\frac{n_B}{n_\gamma}$ is related to the baryon number of the universe B by $\frac{n_B}{n_\gamma} = 1.80\, g_*\, B$. Hence, if $g_* \simeq 3.4$, our mechanism alone can explain the baryon-number-to-photon ratio predicted by nucleosynthesis, $\frac{n_B}{n_\gamma} = (2-7) \times 10^{-10}$, with the $B-L$ breaking scale in the range

$$\eta_{B-L} = (1 \times 10^6 - 2 \times 10^{15}) \text{ GeV}. \tag{5.28}$$

We point out that the result could be better calculated solving Boltzmann equations, which take into account all B, L, and $B+L$ violating interactions and do not neglect the inverse decay rates. Furthermore, the rate of decaying cosmic string loops can be calculated via numerical simulations, which would have to take into account the different regimes of the network evolution which occur during and after the friction dominated era. This may change the allowed value for the $B-L$ breaking scale by a few orders of magnitude. Note also that if CP is not maximally violated, the $B-L$ breaking scale will be shifted towards higher values. Finally, we recall that when a cosmic string loop decays, it also releases massive Higgs bosons ϕ_{B-L} and massive gauge bosons a which can decay into right-handed neutrinos. This process is not taken into account here because the masses of the Higgs and gauge bosons and of the neutrinos are very close to each other and the Higgs and gauge fields can also decay into other particles.

5.5 Conclusions

Unified theories with rank greater or equal to five containing an extra gauge $U(1)_{B-L}$ symmetry which predict heavy Majorana right-handed neutrinos and $B-L$ cosmic strings are good candidates for baryogenesis. $B-L$ cosmic strings are cosmic strings which form at the $B-L$ breaking scale so that the Higgs field used to break $B-L$ is also the Higgs field forming the strings. It is the same Higgs field which gives a heavy Majorana mass to the right-handed neutrinos. There are therefore right-handed neutrinos trapped as transverse zero modes in the core of $B-L$ cosmic strings. The out-of-equilibrium decays of Majorana right-handed neutrinos released by decaying cosmic string loops produce a lepton asymmetry which is then converted into a baryon asymmetry via sphaleron transitions.

The above scenario works for a wide range of parameters. But we have only made a quantitative analysis and therefore a more detailed study needs to be done. Numerical simulations for cosmic string network evolution should be used and full Boltzmann's equations should be solved.

Chapter 6

Conclusions and perspectives

The most appealing feature of particle physics-cosmology is that it may give a reasonable description of the evolution of the universe from very early times until today, and it is a testable theory. The best recent success of the big-bang cosmology is perhaps the perfect black-body spectrum of the cosmic background radiation measured by COBE [15]. The particle physics standard model has been widely tested at high energy colliders, and only one predicted particle, namely the Higgs boson, is missing [22]. There is not yet direct experimental evidence for physics beyond the standard model, but experiments are being undertaken in search of exotic physics such as supersymmetry, neutrino masses, and proton decay. The evidence for new physics may come up soon. The best support for the existence of physics beyond the standard model are baryogenesis, the solar and atmospheric neutrinos problems, which require non-zero neutrinos masses, the high energy gauge coupling constants which require supersymmetry in order to merge in a single point, the necessity of non-baryonic dark-matter, as required by nucleosynthesis, and the requirements of both hot and cold dark-matter to explain structure formation. This thesis is concerned with a small part of this broad subject.

In Chap. 2, the scattering of fermions off cosmic strings arising in a nonsupersymmetric SO(10) GUT model and baryon number violation processes due to the couplings of fermions to GUT gauge bosons in the string core were studied. The elastic cross-sections were found to be Aharonov-Bohm type with a marked asymmetry for left and right-handed fields. The catalysis cross-sections were found to be quite small, unlike previous toy model calculations [36] which suggested that they could be of the order of a strong interaction cross-section. The Callan-Rubakov effect is suppressed, so these strings cannot catalyse proton decay. It is unlikely that grand unified cosmic strings could erase a primordial baryon asymmetry as suggested in Ref. [50]. Also, if cosmic strings and the Aharonov-Bohm effect for cosmic strings were observed, it could help tie down the underlying gauge group.

In Chap. 3, it was argued that supersymmetric SO(10) is a very attractive GUT, from both particle physics and cosmological point of views. SO(10) is the minimal GUT which unifies all kinds of matter and does not include any exotic particle, except a right-handed neutrino.

Also, supersymmetric SO(10) models can solve many of the outstanding problems, like the gauge hierarchy problem, the question of fermion masses and the solar neutrino problem. It was shown that the conditions for topological defect formation in supersymmetric theories follow from those well-known in nonsupersymmetric ones. They are not affected by the presence of supersymmetry. By studying the formation of topological defects in all possible spontaneous symmetry breaking (SSB) patterns from supersymmetric SO(10) down to the standard model gauge group, and also using an inflationary scenario as described in Chap. 4, and by requiring that the model be consistent with proton lifetime measurements, we were able to show that only three of the SSB patterns were consistent with observations. SO(10) can either be broken via $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$, here SO(10) must be broken with a combination of a 45-dimensional Higgs representation and a 54-dimensional one, or via $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_R \times \mathrm{U}(1)_{B-L}$. In both cases, the intermediate symmetry group must be broken down to the standard model gauge group with unbroken matter parity, $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times Z_2$. In supergravity models, the breaking of SO(10) via flipped SU(5) is also possible.

Using the above results, a specific supersymmetric SO(10) model was built. It was described in Chap. 4. The SO(10) symmetry group is broken down to the standard model gauge group, via an intermediate $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_R \times \mathrm{U}(1)_{B-L}$ symmetry. The model gives rise to a false vacuum hybrid inflationary scenario which solves the monopole problem. It is argued that false vacuum hybrid inflation is generic in supersymmetric SO(10) models. It arises naturally from the theory. Neither an external field nor an external symmetry has to be added. In our specific model, cosmic strings form. There are right-handed neutrinos trapped as transverse zero modes in the core of the string. The strings are not superconducting. The model produces a stable LSP and a very light left-handed neutrino which may serve as the cold and hot dark-matter. A mixed cosmic strings and inflationary large scale structure scenario was briefly discussed.

In Chap. 5, a new mechanism for leptogenesis was described. The basic idea is that in unified theories with rank greater or equal to five which contain an extra $\mathrm{U}(1)_{B-L}$ symmetry and predict heavy Majorana right-handed neutrinos, cosmic strings may form as consequence of $\mathrm{U}(1)_{B-L}$ breaking. The Higgs field forming the strings is the Higgs field which breaks $B-L$ and it is also the Higgs field which gives a Majorana mass to the right-handed neutrinos. Due the winding of the Higgs around the strings, there are right-handed neutrinos trapped as transverse zero modes in the core of these strings. When cosmic string loops decay they release these right-handed neutrinos. This is an out-of-equilibrium process. The released neutrinos acquire heavy Majorana mass and decay into massless leptons and electroweak Higgs particles to produce a lepton asymmetry. The lepton asymmetry is converted into a baryon asymmetry via sphaleron transitions. This mechanism could easily be implemented in the supersymmetric SO(10) model described in Chap. 4.

The results of Chapters 4 and 5 confirm that supersymmetric SO(10) is a good GUT candidate. Of course, there are other interesting GUTs, such as the trinification $(\mathrm{SU}(3))^3$, left-right models or E(6). It seems also that only GUTs with rank greater or equal to five make sense phenomenologically. We did not discuss gravity. To include gravity, we should turn to supergravity or superstring models. However, it is interesting to note that both global and local supersymmetric GUTs can be derived from string theories, and hence, since we are working at energies below the string unification scale, it makes sense not to include gravity in our models.

We have studied a mechanism for inflation in global supersymmetric GUT models which is 'natural'. There are many more to be found which will probably lead to different low energy phenomenology. In the model of Ref. [31] for example, topological defects do not form at the end of inflation, but baryogenesis via leptogenesis naturally occurs. Inflation in supergravity models [32] is also the next step to turn to. Inflation in supersymmetric models is usually driven by the VEV of a F-term. Recently, D-term inflation has been proposed [33]. D-term inflation requires further study.

The baryon asymmetry produced by the leptogenesis scenario presented in Chap. 5, has been calculated assuming that the inverse decay rates were negligible and neglecting all other B and L violating processes which may have occured. We have also assumed that the cosmic string network scaling regime was a good approximation to the study of the string network from very early times. A more detailed study can be done, solving Boltzmann's equations and using numerical simulations for cosmic string network evolution which would take into account the different regimes.

The scattering of fermions and the superconducting properties of cosmic strings have been widely studied [7]. It would be a good idea to study cosmic strings arising from a supersymmetric phase transition, before and after supersymmetry breaking. It would be interesting to find signatures of low energy (below the supersymmetry breaking scale) supersymmetric GUT strings, to distinguish them from the usual ones.

Finally, particle physics-cosmology is an intriguing and fascinating field of research and has a very promising future. New experimental results in the search for physics beyond the standard model and new observations of the CBR anisotropy certainly will soon have strong impact on particle physics and cosmology.

Appendix A

Brief review of SO(10)

The fundamental representation of SO(10) consists of 10 generalised gamma matrices. They can be written in an explicit notation, in terms of cross products,

$$\begin{aligned}
\Gamma_1 &= \sigma_1 \times \sigma_3 \times \sigma_3 \times \sigma_3 \times \sigma_3 \\
\Gamma_2 &= \sigma_2 \times \sigma_1 \times \sigma_3 \times \sigma_3 \times \sigma_3 \\
\Gamma_3 &= I \times \sigma_1 \times \sigma_3 \times \sigma_3 \times \sigma_3 \\
\Gamma_4 &= I \times \sigma_2 \times \sigma_3 \times \sigma_3 \times \sigma_3 \\
\Gamma_5 &= I \times I \times \sigma_1 \times \sigma_3 \times \sigma_3 \\
\Gamma_6 &= I \times I \times \sigma_2 \times \sigma_3 \times \sigma_3 \\
\Gamma_7 &= I \times I \times I \times \sigma_1 \times \sigma_3 \\
\Gamma_8 &= I \times I \times I \times \sigma_2 \times \sigma_3 \\
\Gamma_9 &= I \times I \times I \times I \times \sigma_1 \\
\Gamma_{10} &= I \times I \times I \times I \times \sigma_2
\end{aligned} \tag{A.1}$$

where the σ_i are the Pauli matrices and I denotes the two dimensional identity matrix. They generate a Clifford algebra defined by the anti-commutation rules

$$\{\Gamma_i, \Gamma_j\} = 2\,\delta_{ij} \quad i = 1, ..., 10 \; . \tag{A.2}$$

One can define the chirality operator χ, which is the generalised γ_5 of the standard model by

$$\chi = (-i)^5 \prod_{i=1}^{10} \Gamma_i \; . \tag{A.3}$$

In terms of the cross-product notation, χ has the form,

$$\chi = \sigma_3 \times \sigma_3 \times \sigma_3 \times \sigma_3 \times \sigma_3 \; . \tag{A.4}$$

The 45 generators of SO(10) are also given in terms of the generalised gamma matrices

$$M_{ab} = \frac{1}{2i}\,[\Gamma_i, \Gamma_j]\ \ i, j = 1...10\ . \tag{A.5}$$

They are antisymmetric, purely imaginary 32×32 matrices. One can write the diagonal M,

$$\begin{aligned} M_{12} &= \frac{1}{2}\sigma_3 \times I \times I \times I \times I \\ M_{34} &= \frac{1}{2} I \times \sigma_3 \times I \times I \times I \\ M_{56} &= \frac{1}{2} I \times I \times \sigma_3 \times I \times I \\ M_{78} &= \frac{1}{2} I \times I \times I \times \sigma_3 \times I \\ M_{910} &= \frac{1}{2} I \times I \times I \times I \times \sigma_3\ . \end{aligned} \tag{A.6}$$

In SO(N) gauge theories fermions are conventionally assigned to the spinor representation. For N even, the spinor representation is $2^{\frac{N}{2}}$ dimensional and decomposes into two equivalent spinors of dimension $2^{\frac{N}{2}-1}$ by means of the projection operator $P = \frac{1}{2}(1 \pm \chi)$, where 1 is the $2^{\frac{N}{2}} \times 2^{\frac{N}{2}}$ identity matrix. Thus SO(10) has got two irreducible representations,

$$\sigma^{\pm} = \frac{1 \pm \chi}{2} \tag{A.7}$$

of dimension 16. Therefore SO(10) enables us to put all the fermions of a given family in the same spinor. Indeed, since each family contains eight fermions, we can put all left and right handed particles of a given family in the same 16 dimensional spinor. This is the smallest grand unified group which can do so. However, gauge interactions conserve chirality. Indeed,

$$\bar{\psi}\gamma_\mu A^\mu \psi = \bar{\psi_L}\gamma_\mu A^\mu \psi_L + \bar{\psi_R}\gamma_\mu A^\mu \psi_R\ . \tag{A.8}$$

Therefore ψ_L and ψ_R cannot be put in the same irreducible representation. Hence, instead of choosing ψ_L and ψ_R, we chose ψ_L and ψ_L^c. The fields ψ_L and ψ_L^c annihilate left-handed particles and antiparticles, respectively, or create right-handed antiparticles and particles. As mentioned in Sec. 1.2.2, the fields ψ_L and ψ_L^c are related to the fields ψ_R and $\bar{\psi_R}$ by the following relations,

$$\psi_L^c \ \equiv\ P_L\psi^c = P_L C\bar{\psi}^T = C(\bar{\psi}P_L)^T = C\bar{\psi_R}^T = C\gamma_0^T\psi_R^* \tag{A.9}$$

$$\bar{\psi_L^c} \ \equiv\ \psi_L^{c\dagger}\gamma_0 = \psi_R^{*\dagger}\gamma_0^{T\dagger}C^\dagger\gamma_0 = -\psi_R^T C^{-1} = \psi_R^T C \tag{A.10}$$

where the projection operators $P_{L,R} = \frac{1}{2}(1 \pm \gamma_5)$ and C is the usual charge conjugation matrix. For the electron family we get,

$$\Psi_L^{(e)} = (\nu_{(e)}^c\,, u_r^c\,, u_y^c\,, u_b^c\,, d_b\,, d_y\,, d_r\,, e^-\,, u_b\,, u_y\,, u_r\,, \nu_{(e)}\,, e^+\,, d_r^c\,, d_y^c\,, d_b^c)_L \tag{A.11}$$

where the upper index c means conjugate, and the sub-indices refer to quark colour. We find similar spinor $\Psi^{(\mu)}$ and $\Psi^{(\tau)}$ associated with the μ and the τ family respectively:

$$\begin{aligned} \Psi^{(\mu)} &= (\nu_{(\mu)}^c\,, c_r^c\,, c_y^c\,, c_b^c\,, s_b\,, s_y\,, s_r\,, \mu^-\,, c_b\,, c_y\,, c_r\,, \nu_{(\mu)}\,, \mu^+\,, s_r^c\,, s_y^c\,, s_b^c)_L \\ \Psi^{(\tau)} &= (\nu_{(\tau)}^c\,, t_r^c\,, t_y^c\,, t_b^c\,, b_b\,, b_y\,, b_r\,, \tau^-\,, t_b\,, t_y\,, t_r\,, \nu_{(\tau)}\,, \tau^+\,, b_r^c\,, b_y^c\,, b_b^c)_L\ . \end{aligned} \tag{A.12}$$

Appendix B

Scattering amplitude calculations

In this appendix, we give the technical details of the external and internal solutions calculations given in Chap. 2. We also give a discussion of the matching conditions at the core radius.

B.1 The external solution

We want to solve equations (2.19). We set $\partial_t = -i\omega$, where ω is the energy of the electron and take the usual Dirac representation $e_L = (0, \xi_e)$, $e_R = (\chi_e, 0)$, $q_L^c = (0, \xi_q)$ and $q_R^c = (\chi_q, 0)$. We use the usual mode decomposition for the spinors ξ_q, ξ_e, χ_q and χ_e :

$$
\begin{aligned}
\chi_{(e,q^c)}(r,\theta) &= \sum_{n=-\infty}^{n=+\infty} \begin{pmatrix} \chi^n_{1\,(e,q^c)}(r) \\ i\;\; \chi^n_{2\,(e,q^c)}(r)\, e^{i\theta} \end{pmatrix} e^{in\theta} \\
\xi_{(e,q^c)}(r,\theta) &= \sum_{n=-\infty}^{n=+\infty} \begin{pmatrix} \xi^n_{1\,(e,q^c)}(r) \\ i\;\; \xi^n_{2\,(e,q^c)}(r)\, e^{i\theta} \end{pmatrix} e^{in\theta} \, .
\end{aligned}
\tag{B.1}
$$

Then, using the basis,

$$
\gamma^j = \begin{pmatrix} 0 & -i\sigma^j \\ i\sigma^j & 0 \end{pmatrix}
\tag{B.2}
$$

the equations of motion (2.19) become,

$$
\begin{array}{lcccccccccc}
\omega\chi^n_{1,(e,q^c)} & - & (\frac{d}{dr} & + & \frac{n+1}{r} & - & \frac{\tau_{str}^{R\,(e,q^c)}}{r}) & \chi^n_{2,(e,q^c)} & = & 0 \\
\omega\chi^n_{2,(e,q^c)} & + & (\frac{d}{dr} & - & \frac{n}{r} & + & \frac{\tau_{str}^{R\,(e,q^c)}}{r}) & \chi^n_{1,(e,q^c)} & = & 0 \\
\omega\xi^n_{1,(e,q^c)} & + & (\frac{d}{dr} & + & \frac{n+1}{r} & - & \frac{\tau_{str}^{L\,(e,q^c)}}{r}) & \xi^n_{2,(e,q^c)} & = & 0 \\
\omega\xi^n_{2,(e,q^c)} & - & (\frac{d}{dr} & - & \frac{n}{r} & + & \frac{\tau_{str}^{L\,(e,q^c)}}{r}) & \xi^n_{1,(e,q^c)} & = & 0
\end{array} \, .
\tag{B.3}
$$

It is easy to show that the fields $\xi^n_{1,(e,q^c)}$, $\xi^n_{1,(e,q^c)}$, $\chi^n_{1,(e,q^c)}$ and $\chi^n_{2,(e,q^c)}$ satisfy Bessel equations of order $n - \tau_{str}^{R\,(e,q^c)}$, $n + 1 - \tau_{str}^{R\,(e,q^c)}$, $n - \tau_{str}^{L\,(e,q^c)}$ and $n - \tau_{str}^{R\,(e,q^c)}$ respectively. Hence the external solution is,

$$\begin{pmatrix} \xi_{(e,q^c)}(r,\theta) \\ \chi_{(e,q^c)}(r,\theta) \end{pmatrix} = \tag{B.4}$$

$$\sum_{n=-\infty}^{n=+\infty} \begin{pmatrix} & (v_n^{(e,q^c)} Z^1_{n-\tau_{str}^{R\,(e,q^c)}}(\omega r) & + & v_n^{(e,q^c)\prime} Z^2_{n-\tau_{str}^{R\,(e,q^c)}}(\omega r)) & e^{in\theta} \\ i & (v_n^{(e,q^c)} Z^1_{n+1-\tau_{str}^{R\,(e,q^c)}}(\omega r) & + & v_n^{(e,q^c)\prime} Z^2_{n+1-\tau_{str}^{R\,(e,q^c)}}(\omega r)) & e^{i(n+1)\theta} \\ & (w_n^{(e,q^c)} Z^1_{n-\tau_{str}^{L\,(e,q^c)}}(\omega r) & + & w_n^{(e,q^c)\prime} Z^2_{n-\tau_{str}^{L\,(e,q^c)}}(\omega r)) & e^{in\theta} \\ i & (w_n^{(e,q^c)} Z^1_{n+1-\tau_{str}^{L\,(e,q^c)}}(\omega r) & + & w_n^{(e,q^c)\prime} Z^2_{n+1-\tau_{str}^{L\,(e,q^c)}}(\omega r)) & e^{i(n+1)\theta} \end{pmatrix} .$$

The order of the Bessel functions will always be fractional. We therefore take $Z^1_\nu = J_\nu$ and $Z^2_\nu = J_{-\nu}$.

B.2 The internal solution

We get solutions for fields which are linear combinations of the quark and electron fields. Indeed, we get solutions for the fields $\sigma^\pm = \xi_q \pm \xi_e$ and $\rho^\pm = \chi_q \pm \chi_e$. Using the mode decomposition (2.20), the upper components of the fields $\rho^\pm$ and $\sigma^\pm$ are respectively $\rho^\pm_{n1} = \chi^n_{1\,q^c} \pm \chi^n_{1\,e}$ and $\rho^\pm_{n2} = \chi^n_{2\,q^c} \pm \chi^n_{2\,e}$ whilst the lower components are $\sigma^\pm_{n1} = \xi^n_{1\,q^c} \pm \xi^n_{1\,e}$ and $\sigma^\pm_{n2} = \xi^n_{2\,q^c} \pm \xi^n_{2\,e}$ respectively. The equations of motions (2.22) become

$$\begin{array}{ccccccccc} \omega\rho^\pm_{n1} & - & (\frac{d}{dr} & + & \frac{n+1}{r} & \mp & eA') & \rho^\pm_{n2} & = & 0 \\ \omega\rho^\pm_{n2} & + & (\frac{d}{dr} & - & \frac{n}{r} & \pm & eA') & \rho^\pm_{n1} & = & 0 \\ \omega\sigma^\pm_{n1} & + & (\frac{d}{dr} & + & \frac{n+1}{r} & \mp & eA) & \sigma^\pm_{n2} & = & 0 \\ \omega\sigma^\pm_{n2} & - & (\frac{d}{dr} & - & \frac{n}{r} & \pm & eA) & \sigma^\pm_{n1} & = & 0 \end{array} . \tag{B.5}$$

Combining the two first equations of (B.5), one can see that $\rho^\pm_{n1}$ satisfy an hyper-geometric equation giving,

$$\rho^\pm_{n1} = (kr)^{|n|} e^{-ikr} \sum_{j=0}^{n=+\infty} \alpha^\pm_j \frac{(2ikr)^j}{j!} \tag{B.6}$$

where $k^2 = \omega^2 - (eA)^2$, $e = \frac{g}{2\sqrt{2}}$. $\alpha^\pm_{j+1} = \frac{(a^\pm + j)}{(b+p)} \alpha^\pm_j$ with $a^\pm = \frac{1}{2} + |n| \pm \frac{eA(2n+1)}{2ik}$ and $b = 1 + 2|n|$. $\rho^\pm_{n2}$ can be obtained using the coupled equation (B.5.2). We find

$$\rho^\pm_{n2} = -\frac{1}{\omega}(kr)^{|n|} e^{-ikr} \sum_{j=0}^{n=+\infty} \alpha^\pm_j \frac{(2ikr)^j}{j!} \left(\frac{|n| - n}{r} - ik + \frac{j}{r} \pm eA\right) . \tag{B.7}$$

$\sigma^\pm_{n2}$ are also solutions of hyper-geometric equations, and using the coupled equation (B.5.4) we get,

$$\sigma^\pm_{n1} = (kr)^{|n|} e^{-ikr} \sum_{j=0}^{n=+\infty} \beta^\pm_j \frac{(2ikr)^j}{j!} \tag{B.8}$$

$$\sigma^{\pm}_{n2} \;=\; -\frac{1}{\omega}(kr)^{|n|}e^{-ikr}\sum_{j=0}^{n=+\infty}\beta^{\pm}_{j}\frac{(2ikr)^{j}}{j!}\;(\frac{|n|-n}{r}-ik+\frac{j}{r}\pm eA') \tag{B.9}$$

where $k^2 = \omega^2 - (eA')^2$, $\beta^{\pm}_{j+1} = \frac{(c^{\pm}+j)}{(b+p)}\beta^{\pm}_{j}$ with $c^{\pm} = \frac{1}{2} + |n| \pm \frac{eA'(2n+1)}{2ik}$. And the internal solution is,

$$\begin{pmatrix} & \rho^{\pm}_{n1}\, e^{in\theta} \\ i & \rho^{\pm}_{n2}\, e^{i(n+1)\theta} \\ & \sigma^{\pm}_{n1}\, e^{in\theta} \\ i & \sigma^{\pm}_{n2}\, e^{i(n+1)\theta} \end{pmatrix}. \tag{B.10}$$

Therefore the internal solution is giving by a linear combination of the quark and electron fields.

B.3 The matching conditions

The continuity of the solutions at $r = R$ lead to,

$$\begin{aligned}(kR)^{|n|}\, e^{-ikR}\sum_{j=0}^{n=+\infty}\alpha^{\pm}_{j}\frac{(2ikR)^{j}}{j!}& \\ = (v^{q}_{n}\pm v^{e}_{n})J_{n-\tau_R}(\omega R)\;+\;(v^{q'}_{n}\pm v^{e'}_{n})J_{-(n-\tau_R)}(\omega R)&\end{aligned} \tag{B.11}$$

$$\begin{aligned}-\frac{1}{w}\,(kR)^{|n|}\, e^{-ikR}\sum_{j=0}^{n=+\infty}\alpha^{\pm}_{j}\frac{(2ikr)^{j}}{j!}\;(\frac{|n|-n}{R}-ik+\frac{j}{R}\pm eA)& \\ = (v^{q}_{n}\pm v^{e}_{n})J_{n+1-\tau_R}(\omega R)\;+\;(v^{q'}_{n}\pm v^{e'}_{n})J_{-(n+1-\tau_R)}(\omega R)\,.&\end{aligned} \tag{B.12}$$

Nevertheless, we will have discontinuity of the first derivatives. Indeed, inside we have

$$\begin{array}{lllllllll}\omega\rho^{\pm}_{n1} & - & (\frac{d}{dr} & + & \frac{n+1}{r} & \mp & eA') & \rho^{\pm}_{n2} & = 0 \\ \omega\rho^{\pm}_{n2} & + & (\frac{d}{dr} & - & \frac{n}{r} & \pm & eA') & \rho^{\pm}_{n1} & = 0\end{array} \tag{B.13}$$

whereas outside we have

$$\begin{array}{lllllllll}\omega(\chi^{n}_{1,q^c}\pm\chi^{n}_{1,e}) & - & (\frac{d}{dr} & + & \frac{n+1}{r} & - & \frac{\tau^{R\,(e,q^c)}_{str}}{r}) & (\chi^{n}_{2,q^c}\pm\chi^{n}_{2,e}) & = 0 \\ \omega(\chi^{n}_{2,q}\pm\chi^{n}_{2,e}) & + & (\frac{d}{dr} & - & \frac{n}{r} & + & \frac{\tau^{R\,(e,q^c)}_{str}}{r}) & (\chi^{n}_{1,q^c}\pm\chi^{n}_{1,e}) & = 0\end{array}. \tag{B.14}$$

Now,

$$(\chi^{n}_{1,q^c}\pm\chi^{n}_{1,e})^{out} \;=\; \rho^{\pm\, in}_{n1} \tag{B.15}$$

$$(\chi^{n}_{2,q^c}\pm\chi^{n}_{2,e})^{out} \;=\; \rho^{\pm\, in}_{n2} \tag{B.16}$$

giving us the relations for the first derivatives,

$$(\frac{d}{dr}\mp eA)\,\rho^{\pm\, in}_{n2} \;=\; (\frac{d}{dr}-\frac{\tau^{R\,(e,q^c)}_{str}}{R})\,(\chi^{n}_{2,q^c}\pm\chi^{n}_{2,e})^{out} \tag{B.17}$$

$$(\frac{d}{dr}\pm eA)\,\rho^{\pm\, in}_{n1} \;=\; (\frac{d}{dr}+\frac{\tau^{R\,(e,q^c)}_{str}}{R})\,(\chi^{n}_{1,q^c}\pm\chi^{n}_{1,e})^{out}\,. \tag{B.18}$$

Dividing equation (B.11) by equation (B.12) or either replacing equation (B.11) in equation (B.17), we get the following relations

$$\frac{v_n^{q'} \pm v_n^{e'}}{v_n^q \pm v_n^e} = \frac{w\,\lambda_n^{\pm} J_{n+1-\tau_R}(\omega R) + J_{n-\tau_R}(\omega R)}{w\,\lambda_n^{\pm} J_{-(n+1-\tau_R)}(\omega R) + J_{-(n-\tau_R)}(\omega R)} \tag{B.19}$$

where

$$\lambda_n^{\pm} = \frac{\sum_{j=0}^{n=+\infty} \alpha_j^{\pm} \frac{(2ikr)^j}{j!}}{\sum_{j=0}^{n=+\infty} \alpha_j^{\pm} \frac{(2ikr)^j}{j!} \left(\frac{|n|-n}{r} - ik + \frac{j}{r} \pm eA \right)}\ . \tag{B.20}$$

Appendix C

Minimising the superpotential

In this Appendix, we find the true minimum of the superpotential of the model studied in Chap. 4. We calculate the F-terms and find the VEVs of the Higgs fields which correspond to the global minimum.

The full superpotential of the model is given by Eq. (4.13),

$$\begin{aligned} W &= m_A A^2 + m_S S^2 + \lambda_S S^3 + \lambda_A A^2 S + HAH' + m_{H'} {H'}^2 \\ &\quad + m_{A'} {A'}^2 + m_{S'} {S'}^2 + \lambda_{S'} {S'}^3 + \lambda_{A'} {A'}^2 S' \\ &\quad + \alpha \mathcal{S} \overline{\Phi} \Phi - \mu^2 \mathcal{S} \, . \end{aligned} \tag{C.1}$$

where S and S' are 54-dimensional Higgs representations and are traceless second rank symmetric tensors. The S and S' Higgs field must implement the Dimopoulos-Wilczeck meachanism [63]. Therefore, in the 10-dimensional representation of SO(10) they are of the form [80], with appropriate subscripts,

$$< S_{54} >= I \otimes \mathrm{diag}(x, x, x, -\frac{3}{2}x, -\frac{3}{2}x) \tag{C.2}$$

where I is the 2×2 unitary matrix and $x \sim M_{\mathrm{GUT}}$ and

$$< S'_{54} >= I \otimes \mathrm{diag}(x,' x', x', -\frac{3}{2}x', -\frac{3}{2}x') \tag{C.3}$$

where $x' \sim M_{\mathrm{GUT}}$ and are determined by the vanishing condition of the F terms. The Higgs A_{45} and A'_{45} are 45-dimensional representations and must be in the $B-L$ and T_{3R} directions respectively (see Sec. 4.4). Therefore in the 10-dimensional representation of SO(10) A_{45} and A'_{45} are antisymmetric and are given by

$$< A_{45} >= J \otimes diag(a, a, a, 0, 0) \tag{C.4}$$

where $J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ and $a \sim M_{\mathrm{GUT}}$ and

$$< A'_{45} >= J \otimes diag(0, 0, 0, a', a'). \tag{C.5}$$

where $a' \sim M_{\rm GUT}$. The Φ and $\overline{\Phi}$ fields are Higgs fields in the **126** and $\overline{\mathbf{126}}$-dimensional representations. The Φ and $\overline{\Phi}$ fields must break the $\mathrm{U}(1)_X$ symmetry which commutes with SU(5), and thus acquire VEVs in the right-handed neutrino direction. From the vanishing condition for the D terms, $<\Phi>=<\overline{\Phi}>$ and thus, with appropriate subscripts,

$$<\Phi_{126}>_{\nu^c\nu^c}=<\overline{\Phi}_{126}>_{\overline{\nu^c\nu^c}}= d \tag{C.6}$$

where $d \sim M_{\rm G}$. We do not take into account the 10-multiplets which are used to break the standard model gauge group, since we are interesting in what is happening at much higher energies. Their VEVs do not affect the VEVs of the other Higgs fields.

The true vacuum, with unbroken supersymmetry, corresponds to F terms vanishing. Using the same notation for the scalar component than for the superfield, the F terms are given by

$$\begin{aligned}
F_A &= 2m_A A + 2\lambda_A AS, && \text{(C.7)}\\
F_S &= 2m_S S + 3\lambda_S S^2 + \lambda_A A^2,, && \text{(C.8)}\\
F_{A'} &= 2m_{A'} A' + 2\lambda_{A'} A'S' && \text{(C.9)}\\
F_{S'} &= 2m_{S'} S' + 3\lambda_{S'} {S'}^2 + \lambda_{A'} {A'}^2, && \text{(C.10)}\\
F_\Phi &= \alpha\mathcal{S}\overline{\Phi}, && \text{(C.11)}\\
F_{\overline{\Phi}} &= \alpha\mathcal{S}\Phi, && \text{(C.12)}\\
F_{\mathcal{S}} &= \alpha\overline{\Phi}\Phi - \mu^2. && \text{(C.13)}
\end{aligned}$$

Using the VEVs of the Higgs fields given above and also the vanishing conditions for the F terms, we get the following relations, for each term respectively:

$$\begin{aligned}
m_A a + 2\lambda_A a x &= 0, && \text{(C.14)}\\
-m_S x + \frac{3}{4}\lambda_S x^2 + \frac{1}{5}\lambda_A a^2 &= 0, && \text{(C.15)}\\
2m_{A'} a' - 3\lambda_{A'} a' x' &= 0, && \text{(C.16)}\\
-m_{S'} x' + \frac{3}{4}\lambda_{S'} {x'}^2 - \lambda_{A'} {a'}^2 &= 0, && \text{(C.17)}\\
\alpha s d &= 0, && \text{(C.18)}\\
\alpha d^2 - \mu^2 &= 0. && \text{(C.19)}
\end{aligned}$$

where $s = |\mathcal{S}|$. We note that the roles of the 54 dimensional representations S_{54} and S'_{54} are to force the adjoint A_{45} and A'_{45} into $B-L$ and T_{3R} directions. With the VEVs chosen above, see Eqs. (C.2)-(C.6), if $s = 0$ and $d = \frac{\mu}{\sqrt{\alpha}}$ the potential has a global minimum, such that the SO(10) symmetry is broken down to $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_R \times \mathrm{U}(1)_{B-L}$ and supersymmetry is unbroken and we have $x = \frac{2m_A}{3\lambda_A}$ and $x' = \frac{2m_{A'}}{3\lambda_{A'}}$. $a \sim M_{\rm GUT}$, $a' \sim M_{\rm GUT}$, and $\frac{\mu}{\sqrt{\alpha}} \sim M_{\rm G}$, where $M_{\rm G} \sim 10^{15-16}$ GeV and $M_{\rm G} \leq M_{\rm GUT} \leq M_{\rm pl}$ and $M_{\rm pl}$ is the Planck mass $\sim 10^{19}$ GeV.

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